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Brief Papers

Multiple-targets tracking control algorithm for a class of nonlinear systems with feedforward compensations ☆

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A R T I C L E I N F O

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A B S T R A C T

This paper attempts multiple-targets tracking control design for a class of single-input multiple-output (SIMO) nonlinear systems. In order to solve the considered problem, a set of feedforward compensators is preliminarily introduced. The compensators are independent of system states, and their inputs are only target signals and the corresponding derivatives. It is shown that the tracking control law can be efficiently designed under our proposed feedback-controller feedforward-compensator framework. Meanwhile, it is proven that given multiple-target signals can be asymptotically tracked by our designed controllers and compensators. Finally, simulation results are given to show the effectiveness of the theoretical approaches and potential of the proposed design techniques.

1. Introduction

The tracking control problem [1–3] of the following nonlinear system has been thoroughly investigated over the last past a few years:

$$\begin{aligned} &\dot{x}_i = b_i x_{i+1} + f_i(\overline{x}_i), \quad i = 1, 2, \ldots, n-1 \\ &\dot{x}_n = b_n u + f_n(\overline{x}_n), \\ &y = x_1, \end{aligned} \tag{1}$$

where $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i$ are the system states; $b_1, \ldots, b_n$ are nonzero constants; u and y are system input and output respectively; $f_1, \ldots, f_n$ are smooth functions. The controller is designed to make the system output y asymptotically track a prescribed smooth reference signal $y_d(t)$ [4].

As can be seen that, system (1) has a typical up-triangular structure form, whose tracking controller design can be effectively solved by backstepping technique. Interested readers may refer to [5,6] for more details about backstepping. When $f_1, \ldots, f_n$ are completely unknown functions, adaptive fuzzy (or neural-network) controllers have been recently explored to tackle the tracking problem of the systems, see, eg., [7–12] and the references therein. In summary, various types of tracking controllers have been successfully developed in the literature for system (1) whose output can track a desired target signal by designed feedback controllers [13]. It should be pointed out that all the existing results for systems with similar structures of system (1) are only applicable for single-target tracking purpose. It is known that multiple-target tracking is important and necessary in modern complex systems, which however has not been investigated for such class of nonlinear systems so far.

In this paper, we investigate a class of SIMO nonlinear system similar to system (1), where the difference is that more system outputs $y_i = x_i + \phi_i$, $2 \le i \le n$, are considered. Here, $\phi_2, \ldots, \phi_n$ are feed-forward compensators which are independent of system states. The design objective is to design state-feedback controller and feed-forward compensators to make the system outputs $y_1, \ldots, y_n$ asymptotically track the target signals $y_{d1}(t), \ldots, y_{dn}(t)$ respectively. Unlike system (1), the output of our considered system are $y_1, \ldots, y_n$ rather than y_1 only.

To the best of authors' knowledge, the multiple targets tracking control of this type of nonlinear system has not been studied so far. The tracking controller is designed based on backstepping technique. Generally speaking, only x_1 can be controlled to track the desired signal since $x_2, x_3, \ldots, x_n$ need to track their virtual control functions when backstepping technique is applied. Thus, the outputs $y_2, \ldots, y_n$ of the considered system cannot track the target signals without some extra approaches when they have already tracked other signals. To tackle this problem, we propose a method of feed-forward compensation, that is, adding a set of appropriate feed-forward compensations that are independent of the states to the system outputs $y_2, \ldots, y_n$, which enable them to track the targets.

Based on this idea, we start to design those feed-forward compensations. The most important problem is that how to

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make them independent of system states. A key point is that y_1 eventually converge to target y_{d1}, and y_{d1} is known precisely. Thus, y_{d1} can be used instead of y_1 or x_1 when time approaches infinity. Then, state x_2 can be replaced by using the obtained results. Then, through an iteration method, the independent feed-forward compensation can be precisely constructed by only using the function expression of the system, the target signals and their time derivatives.

The purpose of this paper is to solve the multiple targets tracking control problem for a class of SIMO nonlinear systems, which has not been considered so far. The main contributions of this paper are the following: a set of feed-forward compensations is preliminarily proposed to solve the problem of multiple targets tracking when backstepping technique is applied, and the idea of this method can also be transplanted into other more complicated tracking control problem.

2. Problem formulation and preliminaries

Consider a class of SIMO nonlinear system:

$$\begin{aligned}&\dot{x}_i = b_i x_{i+1} + f_i(\overline{x}_i), \quad i = 1,2,\ldots,n-1\\&\dot{x}_n = b_n u + f_n(\overline{x}_n),\\&y_1 = x_1\\&y_i = x_i + \phi_i, \quad i = 2,\ldots,n\end{aligned} \tag{2}$$

where $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i$ are the system states; $b_1,\ldots,b_n$ are nonzero constants; u and $y_1,\ldots,y_n$ are system input and outputs respectively; $f_1,\ldots,f_n$ are smooth functions, $\phi_2,\ldots,\phi_n$ are feed-forward compensators which are independent of system states.

The design objective is to design state-feedback controller and feed-forward compensators make the system outputs $y_1,\ldots,y_n$ asymptotically track the target signals $y_{d1}(t),\ldots,y_{dn}(t)$ respectively.

Remark 1. System (2) can be reduced to the well-known normal form when the output is $y = x_1$, whose studies on feedback control have achieved great developments in the last few years. However, the system model we consider is SIMO type, and there are not any related results on such a type of system so far.

Assumption 1. The tracking targets $y_{d1}(t),\ldots,y_{dn}(t)$ and their time derivatives up to the nth order are continuous and bounded.

3. Control design with backstepping

In this section, the system model without feed-forward compensation is first presented to show the necessity of feed-forward compensation when considering multiple targets tracking control design.

Consider the following SIMO nonlinear system without feed-forward compensation:

$$\begin{aligned}&\dot{x}_i = b_i x_{i+1} + f_i(\overline{x}_i), \quad i = 1,2,\ldots,n-1\\&\dot{x}_n = b_n u + f_n(\overline{x}_n),\\&y_i = x_i, \quad i = 1,\ldots,n\end{aligned} \tag{3}$$

The following coordinate transformations are needed:

$$\begin{aligned}&\xi_1 = x_1 - y_{d1}\\&\xi_2 = x_2 - y_{d2}\\&\quad\vdots\\&\xi_n = x_n - y_{dn}\end{aligned} \tag{4}$$

Then, system (2) is rewritten as

$$\begin{aligned}&\dot{\xi}_i = b_i \xi_{i+1} + f_i(\overline{x}_i) + b_i y_{di} - \dot{y}_{di}, \quad i = 1,2,\ldots,n-1\\&\dot{\xi}_n = b_n u + f_i(\overline{x}_n) - \dot{y}_{dn},\\&y_i = \xi_i + y_{di}, \quad i = 1,\ldots,n\end{aligned} \tag{5}$$

Next, the standard backstepping design procedure will be given.

Let $z_1 = \xi_1$, $z_i = \xi_i - \alpha_{i-1}$, $2 \le i \le n$. Consider the following Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 \tag{6}$$

The derivative of V is obtained as

$$\begin{aligned}\dot{V} = {}& z_1(b_1 z_2 + b_1\alpha_1 + f_1 + b_1 y_{d1} - \dot{y}_{d1}) + \sum_{i=2}^{n-1} z_i(b_i z_{i+1} + b_i\alpha_i + f_i\\&+ b_i y_{di} - \dot{y}_{di} - \dot{\alpha}_{i-1}) + z_n(b_n u + f_n + b_n y_{dn} - \dot{y}_{dn} - \dot{\alpha}_{n-1})\end{aligned} \tag{7}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ are virtual control functions.

Design the virtual control functions $\alpha_1, \alpha_2, \ldots, \alpha_{n-1}$ as

$$\begin{aligned}&\alpha_1 = -\frac{1}{b_1}(f_1 + b_1 y_{d1} - \dot{y}_{d1} + k_1 z_1)\\&\alpha_i = -\frac{1}{b_i}(f_i + b_i y_{di} - \dot{y}_{di} - \dot{\alpha}_{i-1} + k_i z_i + b_{i-1} z_{i-1})\\&\quad 2 \le i \le n-1\end{aligned} \tag{8}$$

where $k_1, k_2, \ldots, k_{n-1} \ge 0$ are design parameters, and for $2 \le i \le n-1$, $\dot{\alpha}_{i-1}$ is defined as

$$\dot{\alpha}_{i-1} = \sum_{s=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_s}(b_s x_{s+1} + f_s) + \sum_{j=1}^{i}\sum_{s=0}^{i-j} \frac{\partial \alpha_{i-1}}{\partial y_{dj}^{(s)}} y_{dj}^{(s+1)}$$

The controller can be designed as

$$u = -\frac{1}{b_n}(f_n - \dot{y}_{dn} - \dot{\alpha}_{n-1} + k_n z_n + b_{n-1} z_{n-1}). \tag{9}$$

Substituting (8) and (9) into (7) yields that

$$\dot{V} = -\sum_{j=1}^{n} k_j z_j^2 \le 0 \tag{10}$$

The design process is completed here.

Eq. (10) indicates that $\lim_{t\to\infty} z_i = 0$, $1 \le i \le n$. Moreover, one has

$$\begin{aligned}&\lim_{t\to\infty} \xi_1 = 0\\&\lim_{t\to\infty} \xi_i - \alpha_{i-1} = 0, \quad 2 \le i \le n\end{aligned} \tag{11}$$

It is easy to see that $\alpha_1, \ldots, \alpha_n \ne 0$ from (8). Hence, it can be obtained that only target y_{d1} can be tracked, and this does not achieve our goal.

In order to make other targets still be tracked, the feed-forward compensators are needed to compensate the system outputs. Next, we state the main result in this paper.

Theorem 1. *Consider the closed-loop system (2) with the controller (9). The multiple targets can be asymptotically tracked by the system outputs when the following feed-forward compensations are applied:*

$$\begin{aligned}&\phi_2 = \frac{1}{b_1}(f_1(x_1)|_{x_1 = y_{d1}} + b_1 y_{d1} - \dot{y}_{d1}),\\&\phi_3 = \frac{1}{b_2}\left\{f_2(\overline{x}_2)|_{x_1 = y_{d1}, x_2 = y_{d2} - \phi_2} + b_2 y_{d2} - \dot{y}_{d2} - \dot{\alpha}_1|_{x_1 = y_{d1}, x_2 = y_{d2} - \phi_2}\right\}, \vdots\\&\phi_n = \frac{1}{b_{n-1}}\left\{f_{n-1}(\overline{x}_{n-1})|_{x_1 = y_{d1}, \ldots, x_{n-1} = y_{d(n-1)} - \phi_{n-1}} - \dot{\alpha}_{n-2}|_{x_1}\right.\\&\quad \left. = y_{d1}, \ldots, x_{n-1} = y_{d(n-1)} - \phi_{n-1} + b_{n-1} y_{d(n-1)} - \dot{y}_{d(n-1)}\right\},\end{aligned} \tag{12}$$

Proof. From (11), one has

$$\lim_{t\to\infty} x_1 = y_{d1}$$