Neurocomputing 194 (2016) 126–134

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Interventional consensus for high-order multi-agent systems with unknown disturbances on coopetition networks

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A R T I C L E I N F O

Article history:
Received 2 August 2015
Received in revised form
26 November 2015
Accepted 31 January 2016
Communicated by Jianbin Qiu
Available online 23 February 2016

Keywords:
Multi-agent systems
Interventional bipartite consensus
Distributed adaptive estimator
Signed graph theory
Dynamical system theory

A B S T R A C T

In this paper, an interventional consensus problem is formulated mathematically with signed graph theory and dynamical system theory. The interaction network associated with a multi-agent system is modeled by a signed graph (called coopetition network in sequel) and the dynamics of each agent is described by a high order differential equation with a nonlinear unknown time-varying disturbance. Then a distributed interaction law is designed for each agent to drive all agents belonging to two competitive subgroups to reach a bipartite consensus on a reference signal, which is generated by an exogenous system (called leader in sequel). Simultaneously, some neural network (NN) based adaptive estimators are proposed to estimate the nonlinear disturbances in the agent dynamics. The convergence of the bipartite consensus is analyzed by using a Lyapunov function method. Finally, some simulation results are presented to demonstrate the formation of the bipartite consensus.

1. Introduction

Recent years have witnessed that collective behavior emerging within a group of autonomous agents (called multi-agent system as well) attracts a lot of attention in different fields, including mathematics, physics, computer science, biology, and control engineering [1–5]. The research on collective dynamics of multi-agent systems not only helps in better understanding the emergence mechanism of complex systems, but also benefits the applications to society, economics and engineering.

For cooperative multi-agent systems, consensus problem has been extensively investigated, where all agents reach an agreement through local coordinative interactions with neighbors [6–8]. However, cooperation and competition are a pair of coexisting relationship between agents in general multi-agent systems. For instance, in the field of biology, [9] found that animals cooperate to look for food or to watch for predators, but compete when resources become limited. In the field of market economics, agents may compete with some of their neighboring agents [10]. It is common to find duopolistic regimes in economic multi-agent systems, where all the agents producing a same product or providing a similar service are often easily split into competitive camps. In the field of social networks, agents can be friends or rivals in contexts like election, games, conflicts, etc. [11]. In the field of game theory, players are divided into two competing teams to maximize their own payoffs. A coopetitive game model was proposed in [12] by virtue of game theory for agents, which cooperate and compete simultaneously in the green economic environment.

Very recently, in the field of system science, an interesting question arises: How the collective dynamics evolves when agents cooperate and compete simultaneously? Altafini proposed a dynamical model by means of the monotone dynamical system theory for each agent and built the cooperative-competitive interaction network associated with the multi-agent system via the signed graph theory [13]. In sequel, the cooperative-competitive interaction network is called coopetition network. Further, the multi-agent dynamics on a coopetition network was described in [14] by a first-order neighbor-based dynamics and a particular collective behavior, namely bipartite consensus, was investigated by using the notation of structural balance. Herein, the bipartite consensus means that all agents reach a final state with identical magnitude but opposite sign. The bipartite consensus problem was also investigated in [15] when the coopetition network is a directed signed graph with a spanning tree. A bipartite consensus problem was considered for a second-order multi-agent system with unknown disturbances in [16]. In [17], a bipartite consensus problem was investigated for a first-order nonlinear multi-agent system with switching network. When the agent dynamics is described by a linear time-invariant system, a static state-feedback control was designed in [18] to guarantee the bipartite consensus.

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http://dx.doi.org/10.1016/j.neucom.2016.01.070
0925-2312/

For dynamical systems with exogenous disturbances, some advanced control strategies have been proposed for linear or nonlinear systems, deterministic, fuzzy or even stochastic systems. For example, some static output feedback H_∞ control protocols were designed for fuzzy affine nonlinear systems [19] and nonlinear hyperbolic PDE systems [20]. An H_∞ filter was designed for Markovian jump linear systems with time-varying delay and partially accessible mode information in [21]. An adaptive neural network (NN) control was proposed to compensate the network-induced delays and packet dropouts for double-layer systems in [22]. For cooperative multi-agent systems with unknown disturbances, some advanced consensus controls have also been developed in the literature. For example, a dynamic output feedback control was designed for first-order multi-agent systems with bounded disturbances in [23]. An adaptive observer-based tracking control was proposed for a second-order leader-follower system with unknown nonlinear dynamics in [24]. A distributed observer-based consensus tracking control was designed for a second-order nonlinear multi-agent system with a directed switched network in [4]. When a first-order agent dynamics had disturbances and uncertainties, a robust adaptive consensus control was designed for each agent by using RBF neural network in [25]. Some adaptive synchronization controllers were proposed in [26] and [27] to deal with the unknown nonlinear dynamics in first- or second-order multi-agent systems with the help of neural network approximation method. A similar adaptive neural control technique was adopted in [28] for a high-order multi-agent system.

In this paper, we focus on the bipartite consensus for a high-order linear multi-agent system on a coopetition network. One basic problem to be addressed is to analyze the formation of bipartite consensus when the agent dynamics suffers from unknown time-varying disturbances. As mentioned above, even though some results have been obtained for bipartite consensus of multi-agent systems, however, to the authors' best knowledge, there exist few related studies to consider bipartite consensus of high-order multi-agent systems with unknown exogenous disturbances. Thus, the contribution of the paper can be twofold. First of all, an exogenous system is introduced to regulate the final bipartite consensus of the multi-agent system. All agents can thus reach bipartite consensus on any desired reference. Secondly, some adaptive neural estimators are proposed to estimate the unknown disturbances. Furthermore, based on the adaptive estimators, a distributed interaction law is designed for each agent to guarantee the interventional bipartite consensus. The convergence of the closed-loop multi-agent system is analyzed by using a Lyapunov function method.

The remainder of this paper is organized as follows. In Section 2, the interventional bipartite consensus problem is formulated and some notations in signed graph theory are presented. In Section 3, a distributed control law together with adaptive neural estimators is proposed for each agent. Then the convergence analysis of the multi-agent system with the proposed control law is analyzed by using a Lyapunov function approach in Section 4. Some simulation results are given to validate the effectiveness of the proposed adaptive bipartite control in Section 5. Finally, some concluding remarks and future research topics are given in Section 6.

Throughout this paper, a vector is denoted by $\mathrm{col}(\cdot)$; the absolute value of a real number is denoted by $|\cdot|$; the Euclidean (Frobenius) norm of a vector (matrix) is denoted by $\|\cdot\|$ ($\|\cdot\|_F$); the trace of a matrix is represented by $\mathrm{tr}(\cdot)$; a diagonal matrix is denoted by $\mathrm{diag}(\cdot)$; the maximum (minimum) singular value of a matrix is $\overline{\sigma}(\cdot)$ $(\underline{\sigma}(\cdot))$. Additionally, $\mathrm{sign}(\cdot)$ denotes the sign function.

2. Problem formulation

2.1. Some preliminaries

We use a signed graph to model a coopetition network. A signed graph $\mathcal{G}=(\mathcal{V},\mathcal{E},A)$, where $\mathcal{V}=\{1,\dots,N\}$ is the set of the nodes of agents, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of the edges, and A is the adjacency matrix of the signed graph. In this paper, we assume that A is a $(-1,0,1)$-matrix. The element a_{ij} of A is attached to the edge $(i,j)\in\mathcal{E}$. If $a_{ij}\neq 0$, then we say that there must be an edge connecting node i and node j. If $a_{ij}>0$, then the interaction between node i and node j is cooperative. In the same way, if $a_{ij}<0$, the interaction is competitive. We define $a_{ii}=0$ for all $i\in\mathcal{V}$. A signed Laplacian matrix associated with the graph $\mathcal{G}$ is denoted by L and defined by

$$L=D-A, \tag{1}$$

where $D=\mathrm{diag}(d_1,d_2,\dots,d_N)$, $d_i=\sum_{j\in N_i}|a_{ij}|$, $N_i=\{j|(i,j)\in\mathcal{E}\}$ is the neighbor set of node i. If another node, labeled 0, is added into $\mathcal{G}$, then one has the augmented graph $\overline{\mathcal{G}}=(\overline{\mathcal{V}},\overline{\mathcal{E}})$, where $\overline{\mathcal{V}}=\{0,1,\dots,N\}$, $\overline{\mathcal{E}}\subseteq\overline{\mathcal{V}}\times\overline{\mathcal{V}}$. Denote the weight between the node 0 and the node i as $b_i\geq 0$. A leader adjacency matrix is defined by a diagonal matrix $B=\mathrm{diag}(b_1,\dots,b_N)\in\mathbb{R}^{N\times N}$. A path with length h is made up of edges with the form $(i_0,i_1),(i_1,i_2),\dots,(i_{h-1},i_h)$ for distinct nodes. A cycle is a path, which starts and ends at the same node. A tree has exactly one parent for every node except the root. A spanning tree is a special tree, where there exists at least one path from the root to any other nodes in the graph. Generally, a cycle can include both positive and negative edges in a signed graph. We say that a cycle is positive, if the product of the weights a_{ij} in the cycle is positive; and negative, otherwise.

Structural balance is a very important concept in the signed graph theory [29,14]. We say that a signed graph is strongly structurally balanced, if all of the cycles in the signed graph are positive. If at least one of the cycles is negative, then we say the graph is structurally unbalanced. If there is no cycle in a signed graph, then we say the graph is vacuously balanced [11]. A signed graph is called structurally balanced [15] when it is strongly structurally balanced or vacuously balanced. Obviously, if a signed graph $\mathcal{G}$ is structurally balanced, then the node set $\mathcal{V}$ can be divided into two subgroups $\mathcal{V}_1=\{1,\dots,N_0\}$ and $\mathcal{V}_2=\{N_0+1,\dots,N\}$, the interactions belonging to the same subgroup are cooperative, while the interactions between them are competitive. Therefore, the adjacency matrix associated with the signed graph $\mathcal{G}$ has the following block form by appropriately reordering the agents

$$A=\begin{pmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}$$

where $A_{11}=A_{11}^T\in\mathbb{R}^{N_0\times N_0}$, $A_{12}=A_{21}^T\in\mathbb{R}^{N_0\times N-N_0}$, and $A_{22}=A_{22}^T\in\mathbb{R}^{N-N_0\times N-N_0}$. It is noted that A_{11} and A_{22} are nonnegative submatrices, while A_{21} is a nonpositive submatrix.

When a coopetition network $\mathcal{G}$ is structurally balanced, it has two antagonistic subnetworks with node sets $\mathcal{V}_1$ and $\mathcal{V}_2$. In order to characterize the relationships among agents belonging to the two subnetworks, a gauge transformation [14,15] is defined by the following diagonal matrix

$$S=\mathrm{diag}(s_1,\dots,s_N)\in\mathbb{R}^{N\times N}, \tag{2}$$

where the diagonal entry $s_i=1$ for $i\in\mathcal{V}_1$ and $s_i=-1$ for $i\in\mathcal{V}_2$. It is not difficult to show that $S^{-1}=S=S^T$. From [15], we have known that

$$SAS=\begin{pmatrix} A_{11} & -A_{12}\\ -A_{21} & A_{22}\end{pmatrix},$$

which means that, through the gauge transformation, the matrix A is similar to a nonnegative adjacency matrix.