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Two novel approaches of UIF design for T-S fuzzy system

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ABSTRACT

This paper researches the state estimation problem for a class of T-S fuzzy system which subject to unknown input. For this set of system, two approaches are studied to design the UIF. A converted system is constructed in the first method to solve the insolubility problem of inequalities because of the existence of the unknown input. The second method establishes filter structure and achieves the purpose of estimating the system states through adopting the direct way. By employing unknown input filter, the paper makes a smooth estimation not only to the system states, but also to the estimate signals and unknown disturbance inputs. At last, a simulation example is used to illustrate the effectiveness of the proposed UIF.

1. Introduction

The estimation of the state variables plays an important role in system analysis and synthesis because the information contained in the state variables of a dynamic model can improve our realization about the system concerned. Hence, state estimation has been an important and interesting problem in the control and fault-tolerant areas and so on for a long time [1–4]. In the past three decades, the state estimation of dynamic systems subjected to both known and unknown inputs has attracted much attention of the researchers' [5–8].

Unknown inputs (UI) can result either from model uncertainty (parameter variations, measurement errors) or due to the presence of unknown external excitation (disturbances, unmodelled dynamics) [9–11]. The existing approaches for estimating the state variables of a T-S fuzzy system which subjects to UI are achieved via two approaches mainly, one is structuring observer and the other is structuring filter (namely UIO and UIF) [12–15]. For many class of systems (fault-tolerant system, fault direction system, switched system, multiple system and so on), estimating the state by structuring UIO is a valid method. In the fields of fault diagnosis, reconstruction, detection, robust control and fault-tolerant control, UIO is utilized for detecting and isolating both actuator and sensor faults [16–22,33]. For switched system with unknown input, Bejarano and Pisano designed a simplified observer for switched linear system which requires an additional structural assumption on the system matrices, and presented a jump observer and discussed the relevant properties of stability [23]. Koenig, Marx, and Jacquet developed a linear matrix inequality technique for the state estimation of discrete-time, nonlinear switched descriptor systems. An observer giving a perfect unknown input decoupled state estimation is proposed [24]. Chadli et al. and Hammouri et al. researched the state estimation problem of chaotic system and state affine system respectively which subjected to unknown input [25,26]. However, for all kinds of systems, in order to guarantee the error systems asymptotically stable, the observer matrices need to meet many limited equations [15,21,27] or there must be many restricted conditions to the order of the observer matrices, which will reduce the conservation [28–30]. Relative to the observer method, there are not many achievements of filter direction, because of its zero block limitation in LMI equation. Zhao, Zhang, and Xing studied the filtering and smoothing for LDTVD systems with norm bounded UI and input noise. The approach can also be extended to linear time-invariant discrete-time descriptor systems and linear discrete-time non-descriptor systems [31]. Charandabi and Marquez proposed an approach to the design of unknown input filters that overcome the structural assumptions in preview researches. The method modifies the plant model in a way that the sampled value of the measurement at time is directly affected by the value of the measurement at time, for some integer determined based on the plant model [32].

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Relative to the [32] complex analysis process, this paper adopts a simpler process with less restricted conditions. One of the greatest difficulties for UIF approach is that the unknown input disturbance is irrelevant to the performance parameter. This leads to zero blocks existence in the LMI when constructing error system which make the LMI no solution. In order to solve this problem, we construct new states which are relevant to the states and vectors of the system and UIF. By defining a new H_∞ performance parameter for the error estimate equation, the solution can be gotten by LMI. This is one of the important contributions of this paper. By utilizing the two analysis methods of UIF, the prescribed disturbance attenuation level is achieved with respect to the unknown input estimation error while guaranteeing the convergence of the filter with a possibly large decay rate of the state estimation error. Meanwhile the main advantage of the proposed approaches boils down to its simplicity. At last, the methods which proposed in the paper have better versatility because we adopted general T-S fuzzy system as the research object.

2. System and filter models

Consider the following non-linear discrete-time system represented by T-S fuzzy dynamic model [34]:

$$\begin{cases} x(k+1)=Ax(k)+B_2d(k)+B_3w(k) \\ y(k)=Cx(k)+D_2d(k)+D_3w(k) \\ z(k)=Hx(k)+F_2d(k) \end{cases} \tag{1}$$

$x(k)\in R^n$ is the state vector; $y(k)\in R^p$ represents the measure output; and $z(k)\in R^q$ is the estimate signal; $d(k)\in R^{m_2}$ is the unknown disturbance input; $w(k)\in R^{m_3}$ represents the state and measurement noise inputs, $d(k)$ and $w(k)$ are assumed to be the arbitrary signal in $l_2[0,\infty)$; with $\Omega=\sum_{i=1}^{r}h_i(\zeta(t))\Omega_i$, Ω means A, B_2, B_3, C, D_2, D_3, H, F. Ω_i means A_i, B_{2i}, B_{3i}, C_i, D_{2i}, D_{3i}, H_i, F_i are the state space matrices of the model with suitable dimension. And $h_i(\zeta(t))\ge 0$, $\sum_{i=1}^{r}h_i(\zeta(t))=1$, $i=1,2,\dots,r$. Here, we assume B_2 is full rank.

In order to estimate the system states, we built the following UIF:

$$\begin{cases} x_F(k+1)=Ax_F(k)+B_2d_F(k)+L(y(k)-y_F(k)) \\ y_F(k)=Cx_F(k)+D_2d_F(k) \\ z_F(k)=Hx_F(k)+F_2d_F(k) \end{cases} \tag{2}$$

where $x_F\in R^n$ is the state vector of the filter; $d_F\in R^{m_2}$ represents the estimation of unknown input disturbance vector, which will follow a stable adaptive law to track the unknown disturbance; $y_F\in R^p$ is the measurement vector which is calculated by using the estimated d_F and x_F; $z_F\in R^r$ represents the estimated vector; and L is the static filter parameter to be designed.

Remark 1. This filter has the same structure and same state space matrices as the original system if the disturbance is zero.

3. UIF design and LMI synthesis conditions

3.1. The first method

In this section, we proposed an LMI-based filter design procedure for discrete-time linear systems subject to unknown input. The following theorem can give a reference to other designers.

Theorem 1. *For the system (1), let γ_1, γ_2 be the l_2 attenuation gains bounding to the effects of unknown input and measurement noise inputs. Then as long as there exit $P>0$, E and invertible matrix G satisfying the following LMI, the H_∞ filter with unknown input will has a solution.*

$$\begin{bmatrix} -P & * & * & * & * \\ 0 & -\gamma_1^2 I & * & * & * \\ 0 & 0 & -\gamma_2^2 I & * & * \\ GA-EC & GB_2-ED_2 & GB_3-ED_3 & -G-G^T+P & * \\ & & & & * \\ H & F_2 & 0 & 0 & -I \end{bmatrix}<0 \tag{3}$$

and $L=G^{-1}E$.

Proof. In order to estimate the states of the fuzzy system which subject to unknown input, we define the following variables as,

$$e(k)=x(k)-x_F(k) \tag{4}$$

$$\overline{d}(k)=d(k)-d_F(k) \tag{5}$$

$$\varepsilon(k)=z(k)-z_F(k) \tag{6}$$

Then $L(y(k)-y_F(k))=L(Ce(k)+D_2\overline{d}(k)+D_3w(k))$, and

$$\begin{aligned} e(k+1)&=x(k+1)-x_F(k+1)=Ae(k)+B_2\overline{d}(k)+B_3w(k) \\ &-L(Ce(k)+D_2\overline{d}(k)+D_3w(k))=(A-LC)e(k)+(B_2-LD_2)\overline{d}(k) \\ &+(B_3-LD_3)w(k) \end{aligned} \tag{7}$$

So, $\varepsilon(k)=z(k)-z_F(k)=He(k)+F_2\overline{d}(k)$. Let

$$\eta(k)=\begin{bmatrix} \overline{d}(k) \\ w(k) \end{bmatrix},$$

then, we plant a new system by using the e, $\overline{d}$, w, ε as follows:

$$e(k+1)=\overline{A}e(k)+\overline{B}\eta(k) \tag{8}$$

$$\varepsilon(k)=He(k)+\overline{D}\eta(k) \tag{9}$$

where $\overline{A}=A-LC$, $\overline{B}=[B_2-LD_2\ \ B_3-LD_3]$ and $\overline{D}=[F_2\ \ 0]$.

Now, we use the following Lyapunov function as,

$$V(k)=e^T(k)Pe(k),\quad P>0 \tag{10}$$

The differential equation of $V(k)$ can be given by:

$$\begin{aligned} \Delta V(k)&=V(k+1)-V(k)=e^T(k+1)Pe(k+1)-e^T(k)Pe(k) \\ &=[\overline{A}e(k)+\overline{B}\eta(k)]^TP[\overline{A}e(k)+\overline{B}\eta(k)]-e^T(k)Pe(k) \end{aligned} \tag{11}$$

In order to establish an H_∞ bound on the effects of the unwanted noise inputs and also the effects of the variations of the unknown disturbance, we define:

$$J\triangleq\sum_{k=0}^{r}[\varepsilon^T(k)\varepsilon(k)-\eta^T(k)\gamma^T\gamma\eta(k)] \tag{12}$$

with $\gamma=\text{diag}[\gamma_1,\gamma_2]$. From (12) and by recalling (1), it can be verified that:

$$\begin{aligned} \Delta V(k)+J&=\Delta V(k)+\varepsilon^T(k)\varepsilon(k)-\eta^T(k)\mu^T\mu\eta(k)=[\overline{A}e(k) \\ &+\overline{B}\eta(k)]^TP[\overline{A}e(k)+\overline{B}\eta(k)]-e^T(k)Pe(k)+\varepsilon^T(k)\varepsilon(k) \\ &-\eta^T(k)\gamma^T\gamma\eta(k)=\nu^T(k)\Xi\nu(k) \end{aligned} \tag{13}$$

where $\nu(k)=\left[e^T(k)\ \ \eta^T(k)\right]$.

Substitute the definition: $\overline{A}=A-LC$, $\overline{B}=[B_2-LD_2\ \ B_3-LD_3]$ and $\overline{D}=[F_2\ \ 0]$ into (13):

$$\begin{aligned} \Xi&=\left[A-LC\quad B_2-LD_2\quad B_3-LD_3\right]^TP\left[A-LC\quad B_2-LD_2\quad B_3-LD_3\right] \\ &+\left[H\quad F_2\quad 0\right]^T\left[H\quad F_2\quad 0\right]+\begin{bmatrix} -P & * & * \\ 0 & -\gamma_1^2 I & * \\ 0 & 0 & -\gamma_2^2 I \end{bmatrix} \end{aligned} \tag{14}$$

Therefore, if the condition $\Delta V(k)+J<0$, namely $\nu^T(k)\Xi\nu(k)<0$ can be found, so the system filtering error can be asymptotically stable towards to zero. Applying the schur complement to (14). Then we pre- and post- multiplying the result by diag$[I\ \ I\ \ I\ \ G\ \ I]$