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Average consensus of multi-agent systems with self-triggered controllers

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ABSTRACT

In this paper the average consensus problem of multi-agent systems under the event-triggered controller is investigated. In event-triggered consensus, an event of each agent occurs only when the state measurement error exceeds a given threshold, and then the agent communicates with its neighbors and updates the control input at the event time. By appropriate event design, discontinuous threshold is employed and thus continuous communication among agents can be avoided to reduce energy consumption in practice. To avoid Zeno behavior, an event/time hybrid trigger is proposed to guarantee asymptotic consensus while excluding Zeno behaviors. Moreover, we develop a self-triggered algorithm to determine the event time instants to further avoid continuous self-state monitoring. Simulation results illustrate the effectiveness of the proposed approach.

1. Introduction

Networked control systems have been extensively investigated in academic research and engineering applications in the past decade [1]. Compared with traditional centralized point-to-point control, networked control is suitable to meet new requirements such as modularity, quick and easy maintenance, and low cost [2]. Various control approaches have been proposed to deal with the constraints such as time-varying delay [3], unknown dynamics [4], affine terms [5], unreliable communication [6] and nonlinear dynamics [7]. An important category of networked systems is the so-called multi-agent system, for which each agent has its own dynamics and a group of agents are connected by a sensing or communication graph [8]. For researchers from control theory and engineering community, the decentralized/distributed coordination control of multi-agent systems has been a hot focus in recent years. Typical research directions in this field include the problem of general/average consensus [9,10], swarming and flocking [11,12], formation control [13], leader–follower tracking [14,4], connectivity control and rendezvous [15,16], and coverage [17,18,5].

In practical multi-agent systems, one of the most important issues is the communication and controller actuation schemes. Digital implementation of sensing, communicating and actuating may call for discontinuous sampling of the state information. Thus how to determine the sampling scheme is critical. Recent advances suggests that in a multi-agent system, each agent may be equipped with small embedded micro-processors and low-cost sensing, communication and actuation modules. These onboard modules usually have only limited energy and abilities. Thus traditional periodic sampling may not be suitable for a practical multi-agent system. Due to this reason, event-triggered control approaches have been proposed [19,20] and then been employed in coordination of multi-agent systems recently [21–25]. In such control approach, the control input of each agent is updated only when the magnitude of the measurement error exceeds a prescribed threshold. It is showed that this control scheme can significantly reduce the number of sampling and thus save much more energy in practice. In [22], to further reduce the communication cost, a self-triggered approach has been proposed to avoid continuous measurement of the neighbors' states. Besides the works on consensus, event-triggered control for some other problems are addressed, including decentralized control over wireless sensor/actuator networks [26] and multi-agent systems with event-based leader–follower tracking [27].

Although significant progress has been achieved in event-triggered coordination of multi-agent systems, how to reduce the system cost and computation still warrants further investigation. From [22], it is noted that by using the self-triggered approach, continuous communication can be avoided. However, the event design is rather complex and the amount of computation is large. Moreover,

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there is no guarantee that no agent will exhibit Zeno behavior. In this work, we have proposed a Zeno-free self-triggered consensus algorithm with discontinuous communication. In the event design, the threshold of the measurement error is discontinuous and only dependent on the transmitted states of the agents at the event time instants. Thus continuous communication is avoided by simple setups. Then by using an event/time hybrid trigger approach, Zeno behavior can be avoided for each agent. Moreover, to further reduce state monitoring, a self-triggered algorithm to determine the event time is developed. The proposed Zeno-free self-triggered consensus algorithm can drive the agent group to achieve average consensus asymptotically. Since it is effective in reducing inter-agent communication and the number of state measurements, the proposed controller may save more energy and significantly lengthen the lifespan of the agents in practical multi-agent systems. Compared with the results in [23] and [28], this work has some major advantages and differences. Firstly, the final consensus values in [23] and [28] are unknown, while in this work it is the average of the initial states of the agents. Secondly, this work uses discontinuous threshold in event design, which naturally excludes continuous inter-agent communication. Thirdly, compared with [28], the event and controller design is different, which causes the design of the event/time hybrid trigger more complex to avoid the Zeno behavior.

The rest of this paper is organized as follows. Section 2 presents the event-triggered average consensus result including the problem formulation, the controller design, the event design, and the consensus analysis. Then the Zeno-free self-triggered control approach as well as the convergence proof are given in Section 3. Section 4 provides the simulation results of the proposed approach to show its effectiveness. Finally the paper is concluded in Section 5.

2. Event-triggered consensus

In this section the event-triggered consensus algorithm with discontinues triggering threshold is presented. Consider a group of N agents in the $\mathbb{R}^n$ space. Each agent is assigned an integer label $i=1,2,\dots,N$ and has the ability of computation, communication and actuation. Assume that the communication between two agents i and j are bidirectional. Then the communication topology is represented by an undirected graph $G=(V,E)$, where $V=\{1,2,\dots,N\}$ is the vertex set, representing the agents, and $E\subset V\times V$ is the edge set, representing the communication links. If agent j can communicate with agent i, then agents i and j are called neighbors. All the neighbor agents of agent i constitute its neighbor set N_i, i.e., $N_i=\{j\,|\,(i,j)\in E\}$. The number of neighbors of agent i is denoted by $n_i=card(N_i)$. The state of agent i is denoted by $x_i(t)\in\mathbb{R}^n$ and the agent kinematic is

$$\dot{x}_i(t)=u_i(t), \tag{1}$$

where $u_i(t)\in\mathbb{R}^n$ is the control input. In many existing works on the general consensus problem [9,10], the consensus protocol is proposed as

$$u_i(t)=-\sum_{j\in N_i}(x_i(t)-x_j(t)). \tag{2}$$

This control law is distributed since each agent only uses the local state information of its neighbors. However, continuous measurement of neighbors' states is required, which may call for large amount of energy consumption and not be applicable in practice. To solve this problem, event-triggered communication and control is employed in this work. Assume that the event time instants of agent i are denoted by

$$t_0^i=0,t_1^i,\dots,t_k^i,\dots,k\in\mathbb{N}. \tag{3}$$

Then each agent measures its state and then transmitted the state information to all the neighbors only at the event time instants. At time t, let the index of the latest event time instant for agent i be

$$k_i(t)=\arg\max_{k\in\mathbb{N}}\{t_k^i\,|\,t_k^i\le t\}. \tag{4}$$

Then the measurement state of agent i at time t is

$$\hat{x}_i(t)=x_i(t^i_{k_i(t)}). \tag{5}$$

Thus once time $t_k{}^i$ comes, agent i will measure the state $x_i(t)$ and transmit $\hat{x}_i(t)$ to all its neighbors for control use. The measurement error for the measurement $\hat{x}_i(t)$ to the realtime state $x_i(t)$ is denoted by

$$e_i(t)=\hat{x}_i(t)-x_i(t). \tag{6}$$

Then the event-triggered controller is proposed as

$$u_i(t)=-\sum_{j\in N_i}(\hat{x}_i(t)-\hat{x}_j(t)). \tag{7}$$

In this controller, agent i updates its control input at its own event time. Its control input will also be updated if it receives any state measurement $\hat{x}_j(t)$ from any neighbor agent.

It is well-known that protocol (2) will solve the consensus problem for the agent group if the communication graph is connected. However, when the event-triggered measurement is adopted, a necessary condition for (7) to solve the consensus problem is that the event time instants should be properly designed. To design appropriate event such that consensus can be achieved, the Lyapunov functional approach is employed. Before presenting the event design, we firstly investigate the final consensus value. Let

$$x(t)=(x_1^T(t),\dots,x_N^T(t))^T \tag{8}$$

and

$$\hat{x}(t)=(\hat{x}_1^T(t),\dots,\hat{x}_N^T(t))^T \tag{9}$$

be the augmented realtime state and measurement state of the agent group, respectively. Then the closed form of the group dynamic can be represented by

$$\dot{x}(t)=-(L\otimes I_n)\hat{x}(t), \tag{10}$$

where L is the Laplacian matrix of the communication graph G. Let the average state of the agent group be

$$\overline{x}(t)=\frac{1}{N}\sum_{i=1}^{N}x_i(t). \tag{11}$$

Consider the time derivative of $\overline{x}(t)$. Since the communication graph is undirected,

$$\begin{aligned}\dot{\overline{x}}(t)&=\frac{1}{N}\sum_{i=1}^{N}\dot{x}_i(t)\\&=\frac{1}{N}\sum_{i=1}^{N}\sum_{j\in N_i}(\hat{x}_j(t)-\hat{x}_i(t))\\&=\frac{1}{N}\sum_{i=1}^{N}\sum_{j\in N_i}(x_j(t)-x_i(t))+\frac{1}{N}\sum_{i=1}^{N}\sum_{j\in N_i}(e_j(t)-e_i(t))=0.\end{aligned} \tag{12}$$

This implies $\overline{x}(t)$ will not change with time. Thus if consensus can be achieved by the agent group, the final consensus value is $\overline{x}(t)$. That is, the agent group will achieve average consensus as time goes to infinity.

Now we will use the Lyapunov functional approach to develop proper event design for the agent group. Consider the following Lyapunov function:

$$\mathbf{V}(t)=\tfrac{1}{2}x^T(t)(L\otimes I_n)x(t). \tag{13}$$

To simplify the notations, denote

$$\hat{z}(t)=(L\otimes I_n)\hat{x}(t) \tag{14}$$