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Leader-following consensus in second-order multi-agent systems with input time delay: An event-triggered sampling approach

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ABSTRACT

This paper analytically investigates an event-triggered leader-following consensus in second-order multi-agent systems with time delay in the control input. Each agent's update of control input is driven by properly defined event, which depends on the measurement error, the states of its neighboring agents at their individual time instants, and an exponential decay function. Necessary and sufficient conditions are presented to ensure a leader-following consensus. Moreover, the control is updated only when the event-triggered condition is satisfied, which significantly decreases the number of communication among nodes, avoided effectively the continuous communication of the information channel among agents and excluded the Zeno-behavior of triggering time sequences. A numerical simulation example is given to illustrate the theoretical results.

1. Introduction

The multi-agent systems (MAS) have attracted great attention due to their broad engineering applications which require multiple robots, vehicles or mobile sensors to work cooperatively to accomplish complex tasks. A particular focus within the control community is the problem of consensus that an agreement has to be reached by all the agents in the whole dynamic network while the information of each agent is locally shared. Numerous contributions have been made on such problems in the literature [1–9] and references therein. An important challenge in multi-agent systems is to design and implement continuous feedback control strategies for control and communication of agents [10]. The traditional digital control techniques are mainly concerned with continuous feedback control strategies under the assumption of unlimited computation and memory resources equipped for each agent in the networks [11]. Nevertheless, an agent in many real networks may have limited resources or it is also expected for the agents to update their control actuation as few as possible for the sake of less time and space complexities. These limitations have resulted in a renewed interest in event-triggered control systems. The centralized and distributed event-triggered control strategies for the consensus of first-order multiple agents connected with undirected networks were presented and the control update was dependent on the state-dependent trigger function [12]. However, the proposed sampling strategy needs updating immediately as long as one of its neighbors completes the sampling, which increases the update frequency of controller. Furthermore, Fan et al. [13] studied a combinational measurement method to event design and improved the basic event-triggered control algorithm for the distributed rendezvous problem of single-integrators. However, their approach requires that the Laplacian matrix of the associated communication topology must be symmetric. In [14], Li. et al. studied the problem of the second-order leader-following consensus by a distributed event triggered sampling strategy. However, after the consensus is reached, the event-triggered control strategy reduces to the traditional continuous-time state feedback control.

Noting that time delay is unavoidable in the real world, the multi-agent systems will be affected by time delay from communication channels and external environment. Then study on the consensus problem for multi-agent systems with time delay is necessary and meaningful, which has been focused on and some related results have been proposed. In addition, when the driving force (acceleration) of multi-agent systems is considered as a control input, each agent should be modeled as a double integrator. In [15], the authors investigated the event-triggered control strategy for the consensus with time delays, where the agents were described by single-integrators. Zhu et al. [16] considered event-triggered leader-following consensus for the linear multi-agent systems with time delay.

In this paper, we considered leader-following consensus in second-order multi-agent systems. We proposed an event-based control strategy with a novel combinational event-triggered

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function, which can not only avoid the continuous communication, but also excludes the Zeno-behavior. Under the proposed controller, necessary and sufficient conditions are presented to ensure a leader-following consensus.

The remainder of this paper is organized as follows. The problem is formulated in Section 2. Some useful lemmas are given and the main theoretical results are derived in Section 3. In Section 4, a numerical example is presented to illustrate the theoretical analysis. Finally, some conclusions and discussions are presented.

The following standard notations will be used throughout this paper: R^n and $R^{n\times m}$ refer to the n-dimensional Euclidean space and the set of all $R^{n\times m}$ real matrices, respectively. Denote $\|\cdot\|$ the Euclidean norm for vectors in R^n or the induced 2-norm for matrices in $R^{n\times m}$. I_n and O_n denote the identity matrix and the zero matrix of order n, respectively. For a real symmetric matrix P, $\lambda_i(P)$ is the ith eigenvalue of P with the ascending ordering of the real parts.

2. Mathematical preliminaries

The communication between the N agents in a interaction network is modeled as a weighted directed graph $G=\{V,E,A\}$, where $V=\{1,2,\ldots,N\}$ is the node set and $A=(a_{ij})_{N\times N}$ is the weighted adjacency matrix of G. A directed edge $(j,i)\in E$ denotes that agent i can obtain information from agent j, or agent j can reach agent i. If there is an edge from agent j to agent i, agent j is called a neighbor of agent i and $a_{ij}>0$; otherwise $a_{ij}=0$. The neighborhood index set of agent i is denoted by $N_i=\{j\in V\,|\,(j,i)\in E\}$. The Laplacian matrix $L=(l_{ij})_{N\times N}$ of a directed graph associated with the adjacency matrix A is defined by $l_{ij}=-a_{ij}\le 0, i\ne 0;\ l_{ii}=\sum_{j=1,j\ne i}^{N} a_{ij}$. $B=diag\{b_1,b_2,\ldots,b_N\}$ is the leader adjacency matrix associated with graph, where $b_i>0$ if there is a directed edge from the leader to the agent i, and $b_i=0$ otherwise.

Consider the following double integrator system of agents:

$$Leader=\begin{cases}\dot{x}_0(t)=v_0(t),\\ \dot{v}_0(t)=0,\end{cases}$$

$$Followers=\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t),\ i=1,2,\ldots,N,\end{cases}\tag{1}$$

where $x_i(t)$, $v_i(t)$ and $u_i(t)$ denote the position, velocity and control input of agent i, respectively. Generally, the input time delay is unavoidable in practice. Thus, we assume that

$$u_i(t)=q_{xi}(t^i_{k_i}-\tau)+q_{vi}(t^i_{k_i}-\tau),\tag{2}$$

where

$$q_{xi}(t)=\sum_{j=1}^{N}a_{ij}\big(x_j(t)-x_i(t)\big)+b_i(x_0(t)-x_i(t)),\tag{3}$$

and

$$q_{vi}(t)=\sum_{j=1}^{N}a_{ij}\big(v_j(t)-v_i(t)\big)+b_i(v_0(t)-v_i(t)),\tag{4}$$

Here, $\tau\ge 0$ denotes the input time delay.

The triggering time sequence $\{t^i_{k_i}\}$ for agent i is declined iteratively by

$$t^i_{k_i+1}=\inf\{t: t>t^i_{k_i}, f_i(t)>0\}\tag{5}$$

where

$$f_i(t)=\|e_{xi}(t)\|+\|e_{vi}(t)\|-\beta_1\|q_{xi}(t^i_{k_i})\|-\beta_1\|q_{vi}(t^i_{k_i})\|-\beta_2 e^{-\gamma(t-t_0)}\tag{6}$$

is said to be the trigger function for some $\beta_1>0$, $\beta_2>0$, $\gamma>0$; $e_{xi}(t)=q_{xi}(t^i_k)-q_{xi}(t)$ and $e_{vi}(t)=q_{vi}(t^i_k)-q_{vi}(t)$. t_0 denotes the initial time. Therefore, $f_i(t)$, $e_{xi}(t)$ and $e_{vi}(t)$ are reset to 0 at $t=t^i_{k_i}$.

In this paper, we assume that each agent can obtain its neighbors' information at $t^i_{k_i}$.

Let $\xi_i(t)=x_i(t)-x_0(t)$, $\eta_i(t)=v_i(t)-v_0(t)$, $\eta(t)=[\eta_1^{\mathrm T}(t),\ \eta_2^{\mathrm T}(t),\ \ldots,\ \eta_N^{\mathrm T}(t)]^{\mathrm T}$, $e_x(t)=\left[e_{x1}^{\mathrm T}(t),\ e_{x2}^{\mathrm T}(t),\ \ldots,\ e_{xN}^{\mathrm T}(t)\right]^{\mathrm T}$, $e_v(t)=\left[e_{v1}^{\mathrm T}(t),\ e_{v2}^{\mathrm T}(t),\ \ldots,\ e_{vN}^{\mathrm T}(t)\right]^{\mathrm T}$. By (1) and (2), we have

$$\begin{cases}\dot{\xi}(t)=\eta(t),\\ \dot{\eta}(t)=-[(L+B)\otimes I_n]\xi(t-\tau)-[(L+B)\otimes I_n]\eta(t-\tau)+e_x(t-\tau)+e_v(t-\tau).\end{cases}\tag{7}$$

Denoting $\varepsilon(t)=\left(\xi^T(t),\eta^T(t)\right)^T$, the system (7) can be further rewritten in the following form:

$$\dot{\varepsilon}(t)=D_1\varepsilon(t)+M\varepsilon(t-\tau)+D_2(e_x(t-\tau)+e_v(t-\tau))\tag{8}$$

where

$$M=\begin{pmatrix}0 & 0\\ -[(L+B)\otimes I_n] & -[(L+B)\otimes I_n]\end{pmatrix},\quad D_1=\begin{pmatrix}0 & I_n\\ 0 & 0\end{pmatrix},$$
$$D_2=\begin{pmatrix}0 & 0\\ 0 & I_n\end{pmatrix}.$$

Definition 2.1. The leader-following consensus is said to be achieved if $\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=0$, $\lim_{t\to\infty}\|v_i(t)-v_0(t)\|=0$ holds for any initial conditions.

Remark 1. Compared with [16], leader-following consensus of second-order multi-agent systems instead of first-order multi-agent systems is considered. Based on the second-order multi-agent systems, a novel combinational event-triggered function is proposed, namely, $t^i_{k_i+1}=\inf\{t: t>t^i_{k_i}, f_i(t)>0\}$, $e_{xi}(t)=q_{xi}(t^i_k)-q_{xi}(t)$ and $e_{vi}(t)=q_{vi}(t^i_k)-q_{vi}(t)$, $f_i(t)=\|e_{xi}(t)\|+\|e_{vi}(t)\|-\beta_1\|q_{xi}(t^i_{k_i})\|-\beta_1\|q_{vi}(t^i_{k_i})\|-\beta_2 e^{-\gamma(t-t_0)}$.

3. Main results

Definition 3.1. The directed graph G is said to have a spanning tree if there is a root node which can reach all the other nodes following the edge directions in graph G.

Lemma 3.1 (*Ren and Beard [17]*). *Assume that there exists a spanning tree in directed graph G. Then the Laplacian matrix L associated with G has eigenvalue 0 with algebraic multiplicity one, and the real parts of all the other eigenvalues are positive, i.e.,* $0=\lambda_1(L)<R(\lambda_2(L))\le\cdots\le R(\lambda_N(L))$.

Lemma 3.2 (*Hu and Hong [18]*). *The matrix $H=L+B$ is positively stable if and only if there exists an directed spanning tree with the root at the node 0 in graph G, where L and B are the Laplacian and leader adjacency matrix of graph G, respectively.*

Lemma 3.3. *Let* $Q=\begin{pmatrix}0_{N\times N} & I_N\\ -(L+B) & -(L+B)\end{pmatrix}$ *and F=L+B, ρ(F) denotes the set of all eigenvalues of F. If F is positive stable i.e., all eigenvalues of F have positive real parts, then* $\max_{\theta\in\rho(Q)}\mathrm{Re}\theta<0$.

Proof. Let θ be an eigenvalue of Q. Then

$$\det(\theta I_{2N}-Q)=\begin{vmatrix}\theta I_N & -I_N\\ (L+B) & \theta I_N+(L+B)\end{vmatrix}=\det\Big(\theta^2 I_N+\theta(L+B)$$
$$+(L+B)\Big)=\prod_{1\le i\le N}\Big(\theta^2+\theta\mu_i+\mu_i\Big)=0$$