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# Compression of hyperspectral remote sensing images by tensor approach

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ABSTRACT

Whereas the transform coding algorithms have been proved to be efficient and practical for grey-level and color images compression, they could not directly deal with the hyperspectral images (HSI) by simultaneously considering both the spatial and spectral domains of the data cube. The aim of this paper is to present an HSI compression and reconstruction method based on the multi-dimensional or tensor data processing approach. By representing the observed hyperspectral image cube to a 3-order-tensor, we introduce a tensor decomposition technology to approximately decompose the original tensor data into a core tensor multiplied by a factor matrix along each mode. Thus, the HSI is compressed to the core tensor and could be reconstructed by the multi-linear projection via the factor matrices. Experimental results on particular applications of hyperspectral remote sensing images such as unmixing and detection suggest that the reconstructed data by the proposed approach significantly preserves the HSI's data quality in several aspects.



## 1. Introduction

Remotely sensed images, which are acquired by the airborne or spaceborne sensors, have been extensively used in earth observation applications. Hyperspectral imaging sensors can collect an image in which each pixel has the contiguous bands of spectra, and these large number of spectral channels provide the opportunity for the detailed analysis of the land-cover materials [1], e.g., endmember extraction [2,3], spectral unmixing [4,5], target detection [6–8], image classification [9–11], and so on . However, as the hyperspectral image (HSI) is intrinsically a data cube which has two spatial dimensions (width and height) and a spectral dimension, numerous researches have indicated that the redundancy from both inter-pixel and inter-band correlation is very high and thus the data cube could be compressed by some algorithms without a significant loss of the useful information for subsequent HSI analysis [12,13].

Generally, image compression technologies can significantly reduce the HSI volumes to a more manageable size for storage and communication. In the literature, most of the existing HSI compression algorithms are transform coding based approaches, e.g., Set Partitioning in Hierarchical Trees (SPIHT) and Set Partitioned Embedded bloCK (SPECK) algorithms [14], the progressive 3-D coding algorithm [15], the 3-D reversible integer lapped transform [16], and the discrete wavelet transform coupled with tucker decomposition [17], etc. Also based on the wavelet transform, Du et al. proposed a series of works on using JPEG 2000 ISO standard for HSI compression, the most important of which are JPEG2000 and Principal Component Analysis (PCA) based HSI compression methods [12,18,19]. As suggested in the aforementioned papers, the transform coding has been proved efficient and practical for HSI compression. However, most of the transform coding related algorithms were originally designed to process 2-D grey-level images, and then extended to 3-D data cube without the consideration of special characteristics of HSI, which might be problematic when the subsequent image analysis is conducted on the reconstructed HSI cube [12,20].

In this paper, we propose a method for compression of the HSIs in a novel point of view, which is based on the multi-dimensional or tensor data processing approach [21–25]. As indicated in some previous works within the hyperspectral imaging area, an HSI data can be intrinsically treated as a 3-order-tensor, by this way the data structure of both the spatial and spectral domains is well preserved [26,27]. For the task of HSI compression, by representing the observed HSI data cube to a 3-order-tensor with two spatial modes and an

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additional spectral mode, we introduce a tensor decomposition technology to decompose the original[2] tensor into a core tensor with same order while much lower dimensionality multiplied by a matrix along each mode, under the umbrella of multi-linear algebra, i.e., the algebra of tensors. Thus, the HSI is compressed to the core tensor, and the reconstructed HSI is actually a low-rank tensor which could be acquired by the multi-linear backward projection via the factor matrices. HSI compression and reconstruction experiments on two public data sets show that the proposed method not only obtains the highest PSNR value, but also significantly preserves the HSI data quality which is benefit for several subsequent image analysis including the endmember extraction, spectral unmixing, and target detection.

The remainder of this paper is organized as follows. In the following section, we give a brief description of related tensor algebra, and then presents the proposed HSI compression algorithm in detail. After that, the experiments are reported in Section 3, followed by the conclusion.

## 2. The proposed HSI compression algorithm

The notations used in this paper are followed by convention in the multi-linear algebra, e.g., vectors are denoted by lowercase boldface and italic letters, such as $\boldsymbol{x}$, matrices by uppercase boldface and italic, such as $\boldsymbol{U}$, and tensors by calligraphic letters, such as $\mathcal{X}$. For a $K$-order-tensor $\mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times \cdots \times L_K}$, where $L_i$ shows the size of this tensor in each mode, and the elements of $\mathcal{X}$ are denoted with indices in lowercase letters, i.e., $\mathcal{X}_{l_1, l_2, \ldots, l_K}$, in which each $l_i$ addresses the $i$-mode of $\mathcal{X}$, and $1 \le l_i \le L_i$, $i \in (1, 2, \ldots, K)$. Unfolding $\mathcal{X}$ along the $i$-mode is defined by keeping the index $l_i$ fixed and varying the other indices, the result of which is denoted as $\mathcal{X}_{(i)} \in \mathbb{R}^{L_i \times \prod_{j \neq i} L_j}$. The $i$-mode product of a tensor $\mathcal{X}$ by a matrix $\boldsymbol{U} \in \mathbb{R}^{J_i \times L_i}$, is a tensor with entries $(\mathcal{X} \times_i \boldsymbol{U})_{l_1, \ldots, l_{i-1}, j_i, l_{i+1}, \ldots, l_K} = \sum_{l_i} \mathcal{X}_{l_1, \ldots, l_K} \boldsymbol{U}_{j_i, l_i}$. The Frobenius norm of a tensor $\mathcal{X}$ is given by $\|\mathcal{X}\| = \sqrt{\sum_{l_1} \cdots \sum_{l_K} \mathcal{X}^2_{l_1, l_2, \ldots, l_K}}$, and the Euclidean distance between two tensors $\mathcal{X}$ and $\mathcal{Y}$ could be measured by $\|\mathcal{X} - \mathcal{Y}\|$. For more detailed information, refer to [21,28–30].

As discussed above, in order to preserve the most representative information of the HSI data, we denote the data cube as a 3-order-tensor $\mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times L_3}$, in which $L_1$, $L_2$, and $L_3$ give the height, width and spectral channels of HSI, respectively. Then, the compressed tensor $\mathcal{C}$ (also known as the core tensor of $\mathcal{X}$) can be acquired by the following multi-linear projection:

$$\mathcal{C} = \mathcal{X} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3 \tag{1}$$

in which $\boldsymbol{U}_1 \in \mathbb{R}^{J_1 \times L_1}$, $\boldsymbol{U}_2 \in \mathbb{R}^{J_2 \times L_2}$, and $\boldsymbol{U}_3 \in \mathbb{R}^{J_3 \times L_3}$ are series of projection matrices and $J_i \le L_i$, $i \in (1, 2, 3)$. By this way, $\mathcal{X}$ is compressed to $\mathcal{C} \in \mathbb{R}^{J_1 \times J_2 \times J_3}$ with the rate of $\prod_{i=1}^3 J_i / \prod_{i=1}^3 L_i$, and the reconstructed tensor could be acquired by the following multi-linear projection:

$$\hat{\mathcal{X}} = \mathcal{C} \times_1 \boldsymbol{U}_1^{\mathrm{T}} \times_2 \boldsymbol{U}_2^{\mathrm{T}} \times_3 \boldsymbol{U}_3^{\mathrm{T}}. \tag{2}$$

The reconstructed tensor $\hat{\mathcal{X}}$ given in (2) is in fact a low-rank tensor, thus the reconstruction error $\mathcal{E}$ could be computed by

$$\mathcal{E} = \mathcal{X} - \hat{\mathcal{X}}. \tag{3}$$

As an effective HSI compression algorithm, we expect that the reconstructed tensor $\hat{\mathcal{X}}$ should be close to the original tensor data $\mathcal{X}$ as much as possible. According to this aspect, the required projection matrices $\boldsymbol{U}_i$, $i \in (1, 2, 3)$ should be optimized by minimizing the Euclidean distance between $\mathcal{X}$ and $\hat{\mathcal{X}}$, which also could be written as the Frobenius norm of $\mathcal{E}$:

$$\arg \min_{\boldsymbol{U}_1, \boldsymbol{U}_2, \boldsymbol{U}_3} \|\mathcal{X} - \hat{\mathcal{X}}\|^2 = \arg \min_{\boldsymbol{U}_1, \boldsymbol{U}_2, \boldsymbol{U}_3} \|\mathcal{E}\|^2. \tag{4}$$

By combining (1), (2) into (4), we have the following optimization of the proposed HSI compression algorithm:

$$\arg \min_{\boldsymbol{U}_1, \boldsymbol{U}_2, \boldsymbol{U}_3} \|\mathcal{X} - \mathcal{X} \times_1 \boldsymbol{U}_1^{\mathrm{T}} \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2^{\mathrm{T}} \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3^{\mathrm{T}} \boldsymbol{U}_3\|^2. \tag{5}$$

Eq. (5) presents the same form with a tensor decomposition technology, i.e., the Tucker decomposition [30,31], which is a form of higher-order PCA and aims to decompose a tensor into a core tensor transformed by a factor matrix along each mode. Thus we abbreviate the proposed method as "TenD" in the rest of this paper. The objective function of (5) could be locally optimized by alternating optimization. The basic idea of this solution comes from the fact that any one of the projection matrix could be simply acquired by an eigenvalue decomposition problem when the remaining two matrices are fixed. So, after initializing $\boldsymbol{U}_i$, $i \in (1, 2, 3)$, the optimal projection matrices along all modes can be acquired iteratively.

Specifically, the projection matrices could be initialized as either identity matrices or arbitrary columnly-orthogonal matrices. In this paper, we suggest to use the higher-order SVD (HOSVD) [30] to find a good starting point for an alternating optimization. Then, the higher-order orthogonal iteration (HOOI) [31] is used to optimize $\boldsymbol{U}_i$, $i \in (1, 2, 3)$ in an iterative way. The detailed procedure for solve Eq. (5) is given below.

**Algorithm 1.** Procedure to solve TenD.

**Input**: Input HSI data $\mathcal{X} \in \mathbb{R}^{L_1 \times L_2 \times L_3}$ and compressed dimensionality in each mode $J_1$, $J_2$ and $J_3$;
**Initialize** $\boldsymbol{U}_i$, $i \in (1, 2, 3)$ using HOSVD;
**repeat**
- $\mathcal{C} = \mathcal{X} \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3$, let $\boldsymbol{U}_1$ be the $J_1$ leading left singular vectors of $\mathcal{C}_{(1)}$;
- $\mathcal{C} = \mathcal{X} \times_1 \boldsymbol{U}_1 \times_3 \boldsymbol{U}_3$, let $\boldsymbol{U}_2$ be the $J_2$ leading left singular vectors of $\mathcal{C}_{(2)}$;
- $\mathcal{C} = \mathcal{X} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2$, let $\boldsymbol{U}_3$ be the $J_3$ leading left singular vectors of $\mathcal{C}_{(3)}$;

**until** Convergence
**Output**: Projection matrices $\boldsymbol{U}_1$, $\boldsymbol{U}_2$ and $\boldsymbol{U}_3$ for HSI compression.

It is worth noting that some representative HSI spectral dimension reduction (DR) algorithms, e.g., PCA and maximum noise fraction (MNF) [32,33], could also perform HSI compression and reconstruction but only in the spectral domain. This branch of approaches considers the HSI data as a set of spectral feature vectors $\boldsymbol{x}_i \in \mathbb{R}^{L_3} | i = [1, \ldots, L_1 L_2]$ in which $L_3$ gives the spectral channels and $L_1 L_2$ is the number of pixels in HSI. Then, the DR algorithm outputs the linear projection matrix $\boldsymbol{U} \in \mathbb{R}^{d \times L_3}$ $(d \le L_3)$ by some certain criterions, e.g., PCA finds the principal components in accordance with the maximum variance of the data and MNF transforms the principal components which are ranked by SNR. Similar to tensor compression (1), the low-dimensional feature representation $\boldsymbol{y}_i \in \mathbb{R}^d$ (here the compression rate is $d/L_3$) is obtained by

$$\boldsymbol{y}_i = \boldsymbol{U} \times \boldsymbol{x}_i, \quad i = [1, \ldots, L_1 L_2] \tag{6}$$

and the reconstructed feature vector could be recovered by the backward projection:

$$\hat{\boldsymbol{x}}_i = \boldsymbol{U}^{\mathrm{T}} \times \boldsymbol{y}_i, \quad i = [1, \ldots, L_1 L_2]. \tag{7}$$

Obviously, Eqs. (6) and (7) consider the feature redundancy in the spectral domain while ignore the cross-domain redundancy of the input HSI data. Correspondingly, the proposed TenD algorithm deals with the HSI data by simultaneously considering both the spatial and spectral domains of the data cube, which can make