Robotics and Autonomous Systems 82 (2016) 15–23

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

# Computing collision-free motions for a team of robots using formation and non-holonomic constraints

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## HIGHLIGHTS

- We compute collision-free motions for a team of non-holonomic robots with formation.
- The resulted paths satisfy all the motion planning constraints.
- The proposed approach, combines techniques from mathematical programming and CAD.

## ARTICLE INFO

*Article history:*
Received 2 May 2015
Received in revised form
18 March 2016
Accepted 22 April 2016
Available online 11 May 2016

*Keywords:*
Motion planning
Multi-robot systems
Formation control
Non-holonomic constraints

## ABSTRACT

This paper focuses on the collision-free motion of a team of robots moving in a 2D environment with formation and non-holonomic constraints. With the proposed approach one can simultaneously control the formation of the team and generate a safe path for each individual robot. The computed paths satisfy the non-holonomic constraints, avoid collisions, and minimize the task-completion time. The proposed approach, which combines techniques from mathematical programming and CAD, consists of two main steps: first, a global team path is computed and, second, individual motions are determined for each unit. The effectiveness of the proposed approach is demonstrated using several simulation experiments.



## 1. Introduction

Robot teams are widely used in industrial environments and warehouses. In many applications, a team of robots is required to meet formations or other constraints to accomplish complex tasks, such as transportation of large objects [1], localization and mapping (SLAM) [2], search and rescue missions [3]. Robot teams are also involved in unmanned aerial vehicle applications [4]. Among these applications, optimal motion planning becomes increasingly important, especially when the demanded task is executed repeatedly [5]. This paper addresses optimal motion planning of a team of robots moving in 2D environments cluttered with narrow passages. The proposed approach enables the control of the team's formation and takes into account the non-holonomic constraints of each robot.

There have been several approaches to maintaining formations of mobile robots [6] including behavior-based methods [7], potential field methods [8], virtual structures [9] and leader–follower approaches [10]. Indeed, several works in robotics and computer animation are related to modeling behaviors like, for example, flocking [11]. The concept of flocking was introduced by Reynolds [11] and describes the behavior of the entities in a team using only local rules for individuals. Later, this technique was extended to include autonomous reactive behavior [12]. These approaches work well in open areas and can generate reasonable natural movements. However, they mainly address the formation-control problem without motion plan optimization, and fail to solve the collision avoidance problem in complicated environments.

Potential field methods [13], construct an artificial force field between the robots of the team. Desired behavior is then created by the combination of a set of potential and velocity fields which guide the team in the desired final configuration. The main limitation of these methods arises from the appearance of local minima in the combined fields, where no descent direction exists for the team to follow.

In the leader–follower approaches [14] some robots have the role of leader and move on predefined trajectories, while the rest of the robots follow them according to a relative posture. An advantage of this approach is that it is relatively easy to implement. A disadvantage, however, is the fact that there is no feedback from the followers to the leaders. If the leaders fail, a new leader must

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http://dx.doi.org/10.1016/j.robot.2016.04.008
0921-8890/

be selected for continued progress. In addition, if a follower fails, it will be left behind and the formation will be broken.

In virtual structure approaches [15], the formation is treated conceptually as a virtual structure with place-holders that represent the desired position for each robot. Using this strategy, it is not possible to consider formations which are time-varying. Moreover, the priority of the robots, either to follow their individual trajectories or to maintain the teams' formation, cannot be changed.

This paper deals with the derivation of feasible collision-free paths for a team of robots satisfying a set of requirements that form a combined global and local motion planning problem. In fact, the first problem (global planning) deals with the determination of the collision-free paths between the start and goal states (position + orientation), while the second problem (local planning) refers to the determination of the optimal motion of the robots inside the team. The proposed approach consists of the following steps: (i) The robots team is modeled as a deformable ellipse. (ii) The global motion planning problem for the team is formulated as a constrained optimization problem which is resolved using a Genetic Algorithm with multiple populations (MPGA) [16]. (iii) The motion of the robots inside the team is designed using a fast geometrically-based technique. (iv) The two motions (global and local) are combined into a final team motion.

The proposed approach has the following properties: (a) Both motion planning and team formations are calculated simultaneously and not in separate stages (like for example in [15]). (b) The generated paths are smooth, collision-free and satisfy the non-holonomic constraints of each individual robot. The current approach does not require post-processing to correct individual movements nor to smooth out corners or other geometric defects. In fact, post-processing algorithms are unable to correct topological problems that occur when global and local motion planning are performed separately.

The remainder of the paper is organized as follows: In Section 2, the motion planning problem is described. Section 3, presents the global motion planning problem for the formation of the robots-team and in Section 4, the optimization algorithm developed for deriving path solutions is described. Section 5, presents the local motion planning of the robots inside the team and Section 6 demonstrates and discusses the efficiency of the proposed approach through multiple experiments. Finally, Section 7 summarizes the contribution of this paper.

## 2. Preliminaries

### 2.1. The motion planning problem

Consider a team of $N$ robots which is moving in a 2D environment cluttered with static obstacles. The robots are requested to move from an initial state to a goal state without colliding with the obstacles, see Fig. 1. The basic assumptions of the environment and the robots movement are as follows:

- Each $m$-robot, $m = 1, \ldots, N$, has the same mechanical structure, i.e., they have the same kinematic model except that the geometric parameters may be different.
- The obstacles have fixed and known geometry.
- Each $m$-robot is requested to move from an initial state $S^m$ to a goal state $G^m$, $m = 1, \ldots, N$. The location and orientation of the start/goal states are defined according to a fixed coordinate system.
- Each $m$-robot is moving only forward with variable velocity in the interval $(0, v_{\max}]$.
- The robots-team follows a formation that is modeled as a deformable ellipse.
- Each $m$-robot is modeled as a car-like robot.

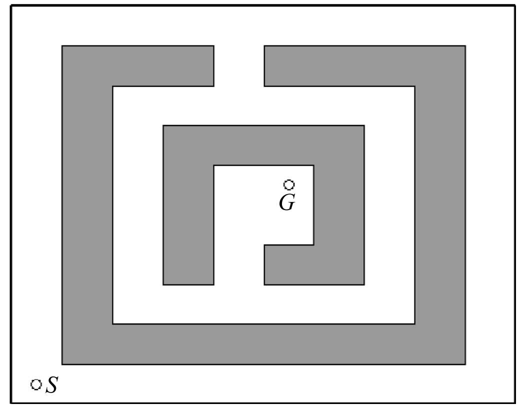


**Fig. 1.** An example of a 2D environment where a team of robots is requested to move from $S$ to $G$.

The motion planning problem addressed in this paper concerns the following three distinct problems which are solved simultaneously:

A. The robots-team should move from a start state to a final goal state following the minimum in length collision-free path (collision avoidance constraint). Note: collision is avoided between robots and obstacles, and among the robots themselves.
B. The robots-team should occupy a constant area and no splits are allowed (formation constraint).
C. Every robot has a maximum allowed velocity and follows a path with an upper-bounded curvature $k_{\max}$ (non-holonomic constraints).

The above requirements form a combined motion planning problem (global and local). In fact, the first problem deals with the determination of the collision-free path between the successive goals of the robots-team and the second problem refers to the determination of the optimal movements inside the team. The overall problem can be characterized as a combinatorial NP-hard problem. Due to the combinatorial explosion, the extraction of exact optimal solutions for NP-hard problems is computationally impracticable. Thus, the reduction of the solution space complexity has a great impact on the final optimum solution. In our approach, this is achieved by using the Bump-Surface method [17] to formulate a search space represented by a single mathematical entity. On the other hand, research on combinatorial explosion based on metaheuristics, such as Genetic Algorithms, can lead to approximate solutions in polynomial time instead of exact solutions that would be at intolerably high cost.

### 2.2. Robot formulation

In order to simplify notation, and without loss of generality, it is assumed henceforth that the 2D environment has unit length in each dimension. Therefore, the entire 2D environment is captured by a normalized workspace $\mathcal{W} = [0, 1] \times [0, 1]$. Each $m$-robot is represented by a car-like robot as it is shown in Fig. 2. It has a rectangular body and its motion is bounded by non-holonomic constraints [18]. The $m$-robot's configuration in the 2D environment is uniquely defined by the triple $(u_1, u_2, \theta) \in \mathcal{W} \times [0, 2\pi)$, where $(u_1, u_2) \in \mathcal{W}$ are the coordinates of the rear axle midpoint **R** with respect to a fixed frame, and $\theta$ represents the orientation of the $m$-robot, as it is shown in Fig. 2. The steering angle $0 \leq \phi \leq \phi_{\max}$ is defined by the main axis of the $m$-robot and the velocity vector at the front-axis midpoint **F**, where $|\phi| = \arctan\left(\frac{l}{\rho}\right) < \frac{\pi}{2}$, $\rho$ is the radius of curvature at point **R** and $l$ is the distance between the midpoints **R** and **F**. Point **G** is the instantaneous center of rotation of the $m$-robot.