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Further studies on impulsive consensus of multi-agent nonlinear systems with control gain error

Tiedong Ma [a,b,*], Liuyang Zhang [b], Zhenyu Gu [b]

[a] Key Laboratory of Dependable Service Computing in Cyber Physical Society (Chongqing University), Ministry of Education, Chongqing 400044, China
[b] College of Automation, Chongqing University, Chongqing 400044, China

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ABSTRACT

In this paper, the impulsive consensus of multi-agent nonlinear systems with control gain error is further studied. The paper indicates that the condition in Theorem 1 of the original paper (Ma et al., 2015) [1] is not correct. To correct the mistake, the new version is given, which deletes one unnecessary coefficient in the inequality condition in the paper Ma et al. (2015) [1]. Moreover, two modified results (Theorems 2 and 3) are given to increase the practicality. Due to the modification of consensus condition, the new simulations are also given correspondingly to verify the effectiveness of the proposed methods.

1. Introduction

In recent years, distributed cooperative control of multi-agent systems has received considerable attention among scientists for its wide application in various fields, such as spacecraft formation flying, sensor networks, and cooperative surveillance [2]. Due to its potential application, the consensus of multi-agent systems has been widely investigated by many researches from various perspectives [2–15]. Generally speaking, the cooperative control of multi-agent systems can be categorized into the leaderless consensus or cooperative regulation problem, and the cooperative tracking problem, where it is desired to synchronize to the dynamics of a leader node. For leaderless consensus, distributed controllers are designed for each node (agent) such that all nodes eventually converge to an unprescribed common value, which may be a constant or time-varying, and is generally a function of the initial states of the agents and the communication network topology [2–6]. For the cooperative tracking problem, a leader node is considered and acts as a command generator that generates the desired reference trajectory [7–15].

Many control methods have been developed to realize the consensus of multi-agent systems, such as observer-based control, adaptive control, pinning control, etc. Among these methods, impulsive control is an efficient method to deal with the dynamical systems which cannot be controlled by continuous control [16–26]. Moreover, one agent receives the information from its neighbors only at the discrete time instants in consensus process, which dramatically reduces the information transmitted between the agents [27–33]. On the other hand, the disturbance often happens when the impulsive controller is given [1,34]. Thus, the consensus of multi-agent systems with impulsive control disturbance (control gain error) is important and significant in real application.

In the paper [1], a sufficient condition including one scalar inequality was given to ensure the consensus of multi-agent nonlinear system with impulsive control gain error. However, the inequality condition (11) of the paper [1] is not correct, and the coefficient $1+m\varphi(t_k)$ is unnecessary. In this paper, we will correct the mistake and give the new version of the consensus condition. Throughout the paper, some common notations and definitions refer to [1] if there is no special explanation.

2. Problem statement

For the sake of readability, some necessary preliminaries and descriptions are given as follows.

* Corresponding author at: Key Laboratory of Dependable Service Computing in Cyber Physical Society (Chongqing University), Ministry of Education, Chongqing 400044, China.
E-mail addresses: tdma@cqu.edu.cn, mtd1118@gmail.com (T. Ma).

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Consider N followers with impulsive control input and control gain error as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + \psi(x_i(t)), t \neq t_k, k \in \mathbb{N}_+ = \{1, 2, ...\}, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = (B_k + \Delta B_k)e_i(t_k) \\ = (b_k + \Delta b_k)\left(\sum_{j \in N_i} a_{ij}(x_i(t_k) - x_j(t_k)) + c_i(x_i(t_k) - x_0(t_k))\right), \ t = t_k, \end{cases} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), ..., x_{in}(t))^T \in R^n$ is the state of node i ($i = 1, \ 2, \ \cdots, \ N$), $\psi: \ R^n \to R^n$ is a continuous nonlinear function, $A \in R^{n \times n}$ is a known constant matrix. The local neighborhood synchronization error for node i denotes by $e_i(t) = \sum_{j \in N_i} a_{ij}(x_i(t) - x_j(t)) + c_i(x_i(t) - x_0(t))$. $B_k = b_k I_n$ is the impulsive control gain matrix. The gain error $\Delta B_k = \Delta b_k I_n$ and nonlinear function $\psi: \ R^n \to R^n$ satisfies Assumptions 1 and 2 respectively.

Assumption 1. The parametric uncertainty Δb_k has the following form,

$$\Delta b_k = m\varphi(t_k)b_k, \tag{2}$$

where $m > 0$ is a known constant, $\varphi(t_k) \in R$ and satisfies $|\varphi(t_k)| \leq 1$.

Remark 1. The uncertainty Δb_k often relates to control gain b_k. In Assumption 1, $\varphi(t_k)$ characterize the change process of Δb_k, and m characterize the magnitude of control gain error. Thus, Assumption 1 is reasonable in real systems.

Assumption 2. Continuous nonlinear function $\psi: \ R^n \to R^n$ satisfies the following condition,

$$\psi(x_1) - \psi(x_2) = \Psi(x_1, x_2)(x_1 - x_2), \tag{3}$$

where $\Psi(x_1, x_2) \in R^{n \times n}$ is the function of vector x_1 and x_2.

Remark 2. Many known nonlinear systems, such as typical chaotic systems (Lorenz system, Chen system, Rössler systems, and unified system), satisfy the condition (3) in Assumption 2.

Consider the dynamics of the leader as

$$\dot{x}_0(t) = Ax_0(t) + \psi(x_0(t)), \tag{4}$$

where $x_0(t) = (x_{01}(t), x_{02}(t), ..., x_{0n}(t))^T \in R^n$ is the state of the leader. We refer to $c_i \geq 0$ as the weight of edge from the leader node to node i ($i \in \{1, ..., N\}$). $c_i > 0$ if and only if there is an edge from the leader node to node i, and $C = \mathrm{diag}\{c_i\} \in R^{N \times N}$.

Subtracting (4) from (1), we can get the impulsive error system described as

$$\begin{cases} \dot{\delta}_i(t) = A\delta_i(t) + \Psi(x_i(t), x_0(t))\delta_i, \\ \Delta\delta_i(t_k) = (b_k + \Delta b_k)e_i(t_k). \end{cases} \tag{5}$$

The compact form of (5) is

$$\begin{cases} \dot{\delta}(t) = (I_N \otimes A)\delta(t) + \overline{\Psi}(x(t), \overline{x}_0(t))\delta(t), \\ \Delta\delta(t_k) = (b_k + \Delta b_k)((L + C) \otimes I_n)\delta(t_k), \end{cases} \tag{6}$$

where

$$\delta(t) = (\delta_1^T(t), \delta_2^T(t), \cdots, \delta_N^T(t))^T, \ \delta_i(t) = x_i(t) - x_0(t),$$

$$\overline{\Psi}(x(t), \overline{x}_0(t)) = \mathrm{diag}\{\Psi(x_1(t), x_0(t)), \cdots, \Psi(x_N(t), x_0(t))\},$$

$$x(t) = [x_1^T(t), \ x_2^T(t), \ \cdots, \ x_N^T(t)]^T \in R^{nN}, \ \overline{x}_0(t) = 1_N \otimes x_0(t) \in R^{nN}.$$

The cooperative tracking problem is to make all follower nodes synchronize to the state trajectory of leader node if for any initial conditions, i.e., $\lim_{t \to \infty} \delta(t) = 0$.

For simplicity, in the rest of the paper, all variables will be denoted without the time argument t, i.e., $x := x(t), x_i := x_i(t)$, $\delta := \delta(t)$, $\delta_i := \delta_i(t)$.

3. Main results

In this section, some new results are derived to correct Theorem 1 in [1].

Theorem 1. *Assume that Assumptions 1 and 2 hold. If there exists $\xi > 1$ such that*

$$(\lambda_A + \lambda_\Psi)(t_k - t_{k-1}) + \ln(\lambda_k \xi) < 0, \tag{7}$$

where λ_A, λ_Ψ and λ_k are the maximum eigenvalue of $A + A^T$, $\overline{\Psi}(x, \overline{x}_0) + \overline{\Psi}^T(x, \overline{x}_0)$ and $(b_k(1 + m\varphi(t_k))(L + C) + I_N)^T(b_k(1 + m\varphi(t_k))(L + C) + I_N)$ respectively. Then the consensus of multiagent systems (1) can be realized under distributed impulsive control.

Proof. Consider a Lyapunov function $V(t)$ as

$$V(t) = \delta^T\delta. \tag{8}$$

For $t \in (t_{k-1}, t_k]$, $k \in \mathbb{N}_+$, the Dini's derivative of $V(t)$ along the trajectory of (6) is given as

$$\begin{aligned} D^+V(t) &= \dot{\delta}^T\delta + \delta^T\dot{\delta} \\ &= \delta^T(I_N \otimes (A + A^T) + \overline{\Psi}(x, \overline{x}_0) + \overline{\Psi}^T(x, \overline{x}_0))\delta \\ &\leq (\lambda_A + \lambda_\Psi)\delta^T\delta \\ &= (\lambda_A + \lambda_\Psi)V(t). \end{aligned} \tag{9}$$

Therefore, one can further get

$$V(t) \leq V(t_{k-1}^+)\exp((\lambda_A + \lambda_\Psi)(t - t_{k-1})). \tag{10}$$

When $t = t_k$, one gets

$$\begin{aligned} V(t_k^+) &= \delta^T(t_k^+)\delta(t_k^+) \\ &= (((b_k + \Delta b_k)((L + C) \otimes I_n) + I_{nN})\delta(t_k))^T(((b_k + \Delta b_k)((L + C) \\ &\quad \otimes I_n) + I_{nN})\delta(t_k)) \\ &= \delta^T(t_k)(((b_k + \Delta b_k)(L + C) + I_N)^T((b_k + \Delta b_k)(L + C) + I_N) \\ &\quad \otimes I_n)\delta(t_k) \leq \lambda_k V(t_k). \end{aligned} \tag{11}$$

Note that the last inequality of (11) uses the fact of Kronecker product, i.e., $\lambda_{\max}(\Pi \otimes I_n) = \lambda_{\max}(\Pi)$, where Π is a square matrix.

Let $k = 1$ in the inequality (10), i.e., for $t \in (t_0, t_1]$, one gets

$$V(t) \leq V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t - t_0)),$$

which leads to

$$V(t_1) \leq V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t_1 - t_0)),$$

and from (11), it further yields

$$V(t_1^+) \leq \lambda_1 V(t_1) \leq \lambda_1 V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t_1 - t_0)),$$

Similarly, for $t \in (t_1, t_2]$,

$$\begin{aligned} V(t) &\leq V(t_1^+)\exp((\lambda_A + \lambda_\Psi)(t - t_1)) \\ &\leq \lambda_1 V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t_1 - t_0))\exp((\lambda_A + \lambda_\Psi)(t - t_1)), \\ &\leq \lambda_1 V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t - t_0)) \end{aligned}$$

In general, for any $t \in (t_k, t_{k+1}]$, it follows from (7) that

$$\begin{aligned} V(t) &\leq V(t_k^+)\exp((\lambda_A + \lambda_\Psi)(t - t_k)) \\ &\leq \lambda_1\lambda_2\cdots\lambda_k V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t - t_0)) \\ &= V(t_0^+)\lambda_1\exp((\lambda_A + \lambda_\Psi)(t_1 - t_0))\lambda_2\exp((\lambda_A + \lambda_\Psi)(t_2 - t_1))\cdots \\ &\quad \times \lambda_k\exp((\lambda_A + \lambda_\Psi)(t_k - t_{k-1}))\exp((\lambda_A + \lambda_\Psi)(t - t_k)) \\ &\leq \frac{1}{\xi^k}V(t_0^+)\exp((\lambda_A + \lambda_\Psi)(t - t_k)) \end{aligned}$$

Since $(\lambda_A + \lambda_\Psi)$ and $(t - t_k)$ are finite constants, and $\frac{1}{\xi^k} \to 0$ as $k \to \infty$(i.e., $t \to \infty$). It is easy to conclude that the global neighborhood error δ is globally exponentially converges to zero, namely, all nodes x_i globally exponentially synchronize to leader node x_0. This completes the proof. □