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Cluster synchronization in complex networks of nonidentical dynamical systems via pinning control

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ABSTRACT

In this paper, we investigate the cluster synchronization problem of complex networks via pinning control. Nodes in the same cluster are governed by the same dynamical function, while the functions for different clusters are different. For the coupling scheme, the effects of the coupling are assumed to be only cooperative, i.e., the coupling matrix is a Metzler matrix with zero row sums, and the connections between the same cluster can be few. For this type of coupling, the main difference of this paper with previous works is that the cluster synchronization is realized by pinning control technique. A simple pinning control strategy is proposed and some sufficient criteria to realize cluster synchronization with both static and adaptive control strength are given. Finally, numerical simulations are presented to verify our theoretical results.

1. Introduction

In the past years, the complex networks have become a hot topic because of their extensive existence in nature and society, such as immune systems, social networks, World Wide Web and electrical power grids. Synchronization, as an interesting phenomenon of complex networks, has attracted many researchers to investigate because of its potential applications in many fields, such as neural networks [1], image processing [2], and secure communication [3]. Researchers have investigated a lot of different synchronization protocols, for example, complete synchronization, cluster synchronization, phase synchronization, lag synchronization, generalized synchronization, and so on.

If all the nodes of the network finally converge to the same trajectory, the complete synchronization is called to be reached, see [4–13]. If the nodes' dynamical behavior is ignored further, the synchronization problem turns into the consensus problem. So far, many graph theories have been introduced to investigate the complete synchronization phenomenon. For example, the master stability function (MSF) method was set up to investigate the stability of the synchronization state in [4]; while the left eigenvector corresponding to the zero eigenvalue of the coupling matrix was used by the authors in [5–8] to investigate synchronization problem; [10,11] studied the consensus problem with or without time delay. Despite these protocols' effectiveness in making the networks achieve synchronization, complex networks cannot synchronize to any given trajectory without external control. In practice, it is difficult to add the controllers on every node of the networks because of the network's scale is usually large. Therefore, to solve the problem, the pinning control strategy is proposed, which means the controllers are just added on partial nodes, see [14–20]. For example, [14] proved rigorously that a complex network can be pinned to synchronize by adding a single controller on just one node.

On the other hand, the cluster synchronization is a more practical phenomenon than the complete synchronization, which is significant in communication engineering [21], biological sciences [22], and so on. The cluster synchronization is characterized by that the nodes in the same cluster achieve a uniform state while the nodes in different clusters have different states. That is to say, the complete synchronization occurs in every cluster, while there is no synchronization between different clusters. Specially, if there is only one cluster in the network, the cluster synchronization problem becomes the complete synchronization.

Investigations have shown that the cluster synchronization depends heavily on the choice of coupling schemes. Generally speaking, there are two main schemes to investigate the cluster synchronization. For the first scheme, nodes in the same cluster only have cooperative connections, while nodes in different clusters can have cooperative or competitive connections, see [23–31]. For example, Ma, Liu and Zhang were the first to propose such a new coupling scheme to realize cluster synchronization in [23]; the

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authors in [24] generalized the scheme to asymmetric matrices to realize the cluster synchronization; [25] investigated the cluster synchronization under pinning control with symmetric coupling matrix; [26] generalized the coupling matrix to be asymmetric and the pinning control can be periodically intermittent; [27] studied the cluster synchronization under nonlinear coupling and periodically intermittent pinning control. All these works were based on the construction of the coupling matrix, which played a key role in the analysis of cluster synchronization problem.

The other scheme for the cluster synchronization is to assume that all the coupling nodes only have cooperative connections, i.e., the coupling matrix is a Metzler matrix with zero row sums, where a Metzler matrix is a matrix in which all the off-diagonal components are nonnegative. For example, in [32,33], for a given nearest-neighborhood network with zero-flux or periodic boundary conditions, the authors proposed an effective method to determine some possible states of cluster synchronization and their stability; in [34], for linearly and symmetrically coupled networks, the authors discussed the connection between the cluster synchronization and the complete synchronization by analyzing the invariant synchronization manifold and proposed several criteria for the global attractivity of the invariant synchronization manifold. Recently, [35] studied the cluster synchronization in networks of coupled nonidentical systems and indicated that the common intercluster coupling condition and the intracluster communication played key roles for cluster synchronization. An interesting phenomenon is reported, the network can realize cluster synchronization even if there is no connections between nodes of the same cluster. Refs. [36,37] studied the cluster synchronization and cluster consensus for discrete-time coupled systems respectively, and [38] investigated the cluster consensus problem for continuous-time coupled systems. Ref. [39] showed how different mechanisms may lead to clustering behavior in connected networks consisting of diffusively coupled agents by considering self-dynamics, delays, both positive and negative couplings.

Until now, most existing works only discuss the cluster synchronization by mutual coupling, however, in some cases, the final synchronization trajectories should also be considered. That is to say, if we want the nodes finally converge to some targets, the external control should be added. Moreover, for constant coupling strength, the cluster synchronization may not be realized, while by adding external control, the cluster synchronization can be achieved, see [40–42]. However, all these works realize the synchronization by adding the controllers on all clusters, that is to say, any cluster should be finally synchronized to these given targets.

To our best knowledge, there are few works discussing the cluster synchronization by adding the external controllers on just partial clusters. Motivated by the above discussions, we will investigate the cluster synchronization of complex networks via pinning control. There are three contributions in this paper: (1) a new network model for cluster synchronization is set up, only partial clusters are controlled while others are not; (2) criteria are obtained by using pinning control and synchronization techniques; (3) a new adaptive scheme for coupling strength to realized cluster synchronization is proposed and rigorously proved.

The rest of this paper is organized as follows. In Section 2, the network model is described and some necessary definitions, lemmas, assumptions and notations are given. In Section 3, we investigate the cluster synchronization problem with pinning control and some sufficient criteria are proposed to guarantee the cluster synchronization. The adaptive technique is also applied on the coupling strength and its effectiveness is rigorously proved in Section 4. In Section 5, some numerical simulations are presented to show the validity of theoretical results. Finally, this paper is concluded in Section 6.

2. Preliminaries

The graph $\mathcal{G}$ can be denoted by a double set $\{\mathcal{V},\mathcal{E}\}$, where $\mathcal{V}$ represents the vertex set numbered by $\{1,\dots,N\}$, and $\mathcal{E}$ denotes the edge set with $e(i,j)\in\mathcal{E}$ if and only if there is an edge from vertex j to i. Assume the set of nodes in the network can be divided into m clusters, i.e., $\{1,\dots,N\}=\mathcal{C}_1\cup\mathcal{C}_2\cup\cdots\cup\mathcal{C}_m$ where

$$\mathcal{C}_1=\{1,\dots,r_1\},\mathcal{C}_2=\{r_1+1,\dots,r_2\},\dots,\mathcal{C}_m=\{r_{m-1+1},\dots,N\}. \quad (1)$$

The network of coupled dynamical systems is defined on the graph $\mathcal{G}$. The individual uncoupled system on the vertex i is denoted by an n-dimensional ordinary differential equation

$$\dot{x}_i(t)=f_k(x_i(t)),$$

for $\forall i\in\mathcal{C}_k$, where $x_i=[x_i^1,\dots,x_i^n]^\top$ is the state variable vector on vertex i and $f_k(\cdot):R^n\to R^n$ is a continuous vector-valued function. Each vertex in the same cluster has the same individual node dynamic. It should be emphasized that nodes in different clusters are nonidentical, which can guarantee that the trajectories are apparently distinguishing when cluster synchronization is reached.

The interaction among vertices is denoted by linear diffusion terms. That is to say, the dynamical behaviors of the network can be described by

$$\dot{x}_i(t)=f_k(x_i(t))+c\sum_{j=1}^{N}a_{ij}\Gamma(x_j(t)-x_i(t)),\quad i\in\mathcal{C}_k,\ k=1,\dots,m \quad (2)$$

In [35], the cluster synchronization of the above model has been investigated, but the synchronization heavily depends on the value of the coupling strength c. So we will investigate the linearly coupled systems by adding some external controllers in order to realize the cluster synchronization with a small value of c.

Before our discussion, we first present some useful definitions, notations and lemmas, which will be used throughout the whole paper.

We make the following assumption for function $f_k(\cdot)$.

Assumption 1 (*QUAD condition, Lu and Chen [5,6], Liu and Chen [7,8]*). The function $f_k(\cdot),k=1,2,\dots,m$, is said to satisfy the QUAD condition, denoted as $f_k(\cdot)\in QUAD(\alpha,\eta)$. That is, if for some $\alpha\in R$, there exists $\eta>0$ such that

$$(x-y)^\top[f_k(x)-f_k(y)-\alpha\Gamma(x-y)]\le-\eta(x-y)^\top(x-y) \quad (3)$$

holds for all $x,y\in R^n$.

Definition 1 (*Liu and Chen [26]*). Matrix $A=[a_{ij}]_{i,j=1}^N$ of order N is said to belong to class $A1$, denoted as $A\in A1$, if

1. $a_{ij}\ge0, i\ne j$, $a_{ii}=-\sum_{j=1,j\ne i}^N a_{ij}$, $i=1,\dots,N$,
2. A is irreducible.

Furthermore, if $A\in A1$ and $a_{ij}=a_{ji},\ i\ne j$, then we say $A\in A2$.

Lemma 1 (*Lu et al. [35]*). *If a matrix $A_{N\times N}\in A2$, its eigenvalues are all real and can be sorted as*

$$0=\lambda_1(A)>\lambda_2(A)\ge\lambda_3(A)\ge\cdots\ge\lambda_N(A). \quad (4)$$

Definition 2 (*Wu and Chen [34]*). Matrix $A=[a_{ij}]\in R^{N_1\times N_2}$ is said to belong to class $A3$, denoted as $A\in A3$, if its each row-sum is equal, i.e., $\sum_{j=1}^{N_2}a_{i_1j}=\sum_{j=1}^{N_2}a_{i_2j}$, $i_1,i_2=1,\dots,N_1$.

Now, using the above definitions of matrices, we can define a new type of coupling matrix A for the following cluster synchronization analysis.

Definition 3 (*Wu and Chen [34]*). Suppose $A\in R^{N\times N}$, the indexes $\{1,\dots,N\}$ can be divided into m clusters as defined in (1), and the