Neurocomputing 158 (2015) 26–31

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Improvement of consensus convergence speed for linear multi-agent systems based on state observer☆

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ARTICLE INFO

Article history:
Received 9 October 2014
Received in revised form
16 January 2015
Accepted 5 February 2015
Available online 18 February 2015

Keywords:
Multi-agent systems
Consensus
Convergence speed
State observer

ABSTRACT

A novel consensus protocol is presented for linear multi-agent systems(MASs) in which all the agents have the same continuous-time linear dynamics with high dimension. A fast convergence speed can be obtained by using this protocol for the MASs where each agent gets its own state through the full-order observer. Every agent can also obtain the states of its neighbors' and its instantaneous neighbors' to reach consensus. Necessary and sufficient conditions of consensus for the MASs are proposed to improve the convergence speed. Finally, the corresponding convergence speeds of different protocols are showed by numerical simulation, and extended to directed topology.

1. Introduction

Many consensus protocols are proposed to solve a consensus problem of Multi-Agent systems(MASs). The history of consensus can date back to 1960s, from then on, distributed computing is developed in the field of computer science. In 1987, Boid model [1] is the first proposed to describe the behavior of natural populations by some simple rules. In 1995, a simulation is presented to simulate the consensus of a group of particles by Vicsek [2]. Olfati and Murray [3] proposed an average consensus protocol, which lays the foundation of the consensus problem, and discusses the importance of convergence speed. The convergence speed depends on the algebraic connectivity of the topology of the MASs. The protocol is also extended to MASs with symmetric time delay. Ren and Atkins [4] give a protocol to solve the consensus problem for the second-order MASs, and apply to control the height consensus of multiple vehicle systems. Song et al. [5] designs a protocol to achieve leader-follower consensus for MASs. With a detailed study, some complex systems are studied. Hou et al. [6] studies the systems based on the group combined information, which means a single agent is effected by two groups. At the same time, in [7], Lin et al. brings complex value Laplacian matrix in MASs where the complex is used to study the leader-follower formation problem. Tan et al. [8] investigates the problem of consensus for discrete-time networked multi-agent systems (NMASs), where information is exchanged through the network with a communication delay, and a novel distributed protocol is proposed to compensate the delay actively. Li and Ren [9] design an adaptive consensus protocol for MASs with general linear and Lipschitz nonlinear dynamics. Cheng et al. [10] considers continuous-time double-integrator MASs with measurement noises under fixed topologies, and it is proved that in the noisy communication environment the average consensus can be achieved.

In the practice, the convergence speed is important to MASs. Researchers have worked on it for a long time. Based on the finite-time stability theorem, Wang et al. [11] proposes a finite-time consensus protocol for the first-order MASs. [12] studies the finite-time consensus problem of multiple nonholomic mobile agents. Lin et al. [13] discusses a class of discrete-time switched linear systems, and uniform finite-time stability and feedback stabilization of the systems were studied. Khoo et al. [14,15] design a new distributed control protocol by terminal sliding mode for MASs, which is based on the finite-time stability theorem. [11–20] are all worked on how to accelerate the convergence speed by finite-time consensus protocol. Jin and Murray [21] propose multi-hop relay protocols on the current "nearest neighbor rules" consensus protocols, in order to achieve a faster consensus seeking. In [22], the weighted average prediction(WAP) protocol is introduced into the existing consensus protocol by Fang. WAP can improve not only the robustness against communication delay but also

☆This work was supported by the National Natural Science Foundation of China under grant (61374083), Science and Technology Department Project of Zhejiang Province under Grant (2014C31082) and 521 Talent Project of Zhejiang Sci-Tech University.

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http://dx.doi.org/10.1016/j.neucom.2015.02.009
0925-2312/

the convergence speed. In [23,24], state feedback controller is constructed to make the system stable. Wu et al. [25,26] have studied the neural networks and linear matrix inequality toolbox is used to get the stability condition.

Motivated by these existed results, an observer to compensate for the unmeasurable states is designed. And multi-hop network is applied to high-order MASs with balanced digraph topology by adding virtual edges in this paper. The necessary and sufficient conditions for consensus problem are presented. The effectiveness of the proposed protocol is verified by numerical simulation. And from the result of numerical simulation, it can be obtained that our protocol generates a fast convergence rate. In this paper, full-order observer and multi-hop network are combined to modified the convergence speed of consensus protocol.

2. Preliminaries

Consider the MASs with$N$ agents. Directed graph can be used to model the communication topology of the MASs. A directed graph $G=(V,E,A_g)$, where $E=\{e_{ij}=(v_i,v_j):v_i,v_j\in V\}$,$V=\{v_1,v_2,\dots,v_N\}$, $A_g=(a_{ij})\in R^{N\times N}$denote the edge set, vertex set and a weighted adjacency matrix, respectively. $e_{ij}\in E$, if and only if there is information flows from agent v_j to agent v_i, node v_j is called the parent node, node v_i is called the child node. The neighbor set of vertex $v_i$is denoted by $N_i=\{j\in V\,|\,e_{ji}\in E\}$. a_{ij}, $a_{ii}=0, a_{ij}>0\Leftrightarrow j\in N_i$, denotes the weighted non-negative adjacency elements. And $\deg_{out}(i)=\sum_{j=1}^{N}a_{ji}$ is called the out-degree of nodev_i, $\deg_{out}(i)=\deg_{in}(i)$ is called the out-degree of node v_i. Then G is a balanced digraph, if $\deg_{out}(i)=\deg_{in}(i)$, for any $i, i\in\{1,2,\dots,N\}$. The Laplacian matrix L_g of a digraph G is defined by $L_g=D_g-A_g$, where $D_g=diag(\deg_{in}(1),\deg_{in}(1),\dots,\deg_{in}(N))$ is the in-degree matrix of the digraph.

Notation:$1_n=(1,1,\cdots1)\in R^n$, $0_n=(0,0,\dots,0)\in R^n$. $I_n\in R^{n\times n}$ denote the identity matrix. $\otimes$ is the Kro -necker operator. If X is a matrix, X^Tdenote its transpose, X^H is the conjugate transpose, $R(X)$ is the range of X, and $N(X)$ is the kernel space of X. For $X\in R^n$, $\dim(X)$ denotes the dimension of X.

3. Main results

The i-th agent is described by the following continuous-time linear dynamics

$$\begin{cases}\dot{x}_i=Ax_i+Bu_i\\ y_i=Cx_i\end{cases}\tag{1}$$

where $x_i\in R^n$ is the state, $u_i\in R^p$ is the input, and $y_i\in R^q$ is the output of the agentv_i. So $A\in R^{n\times n}$, $B\in R^{n\times p}$, and $C\in R^{q\times p}$are the state, input and output matrices, respectively. It is assumed that all agents in the whole paper have the same linear time invariant dynamical model. It implies that A,$B$and $C$are all constant matrices. A simple and common consensus protocol can be described as:

$$u=\sum_{j=1}^{N}a_{ij}(x_j-x_i)\tag{2}$$

Based on the above protocol, [3] proposed a static output feedback protocol, the local static output error can be described:

$$z_i=\sum_{j\in N_i}\left(y_i-y_j\right), i=1,2,\dots,N.\tag{3}$$

And [27] proposed a consensus protocol based on the static output error:

$$u_i=Kz_i=K\sum_{j\in N_i}\left(y_i-y_j\right), i=1,2,\dots,N\tag{4}$$

Convergence speed is a very important performance index to a consensus protocol, and some researchers have studied the finite-time consensus to improve the convergence speed. Nevertheless, there are few results to consider the measurement cost of the systems. And it is difficult to measure the state of the agents, or due to the cost of the measurement in the practice. [28] has proposed a protocol based on the state observer to solve the problem in infinite time field:

$$\begin{cases}u_i=K_1z_i+\sum_{j=1}^{N}a_{ij}K_2(z_j-z_i)\\ \dot{z}_i=(A+LC)z_i+Bu_i-Ly_i\end{cases}\tag{5}$$

In order to improve the convergence speed, a modified consensus protocol based on (5) is proposed in (6) which can be formulated:

$$\begin{cases}u_i=K_1z_i+K_2\sum_{j=1}^{N}a_{ij}((z_j-z_i)+\sum_{k=1}^{N}a_{jk}(z_k-z_i))\\ \dot{z}_i=(A+LC)z_i+Bu_i-Ly_i\end{cases}\tag{6}$$

wherea_{ij} is the i-th row and j-th column element of the adjacency matrix A_g. z_i is the output of the full-order observer. K_1, K_2 are weighted constant matrices, L is the output feedback gain of the observer. Each vertex sends to its parent vertices not only its own state, but also a collection of its instantaneous neighbors' states. N_i denotes the set which contains all neighbors of node v_i, and N_{ij} denotes the set which contains all neighbors of N_i. So the nodes of N_{ij} have information flow to the nodes of N_i, or node v_i can get the information of node v_k, $k\in N_{ij}$ indirectly. It can be treated that there exists virtual edges between node v_i and node v_k. So there exists a virtual balanced digraph $\tilde{G}$. Let $\tilde{A}_g$, $\tilde{L}_g$ be the virtual adjacency matrix and the virtual Laplacian matrix of the virtual balanced digraph $\tilde{G}$.

Combining (1) – (6), we can obtain

$$\begin{cases}\dot{x}_i=Ax_i+BK_1z_i+BK_2\sum_{j=1}^{N}a_{ij}((z_j-z_i)+\sum_{k=1}^{N}a_{jk}(z_k-z_i))\\ \dot{z}_i=(A+LC)z_i+BK_1z_i+BK_2\sum_{j=1}^{N}a_{ij}((z_j-z_i)+\sum_{k=1}^{N}a_{jk}(z_k-z_i))-LCx_i\end{cases}\tag{7}$$

A_g is the adjacency matrix of a balanced digraph, so we can get its Laplacian matrix L_g easily:

$$l_{ij}=\begin{cases}\sum_{j=1,j\neq i}^{N}a_{ij}, i=j\\ -a_{ij}, \quad i\neq j\end{cases}\tag{8}$$

Where l_{ij} is the i-th row and the j-th column element of L_g. $\tilde{A}_g,\tilde{L}_g$ be the virtual adjacency matrix and the virtual Laplacian matrix of the virtual balanced digraph $\tilde{G}$. It is obviously that:

$$\tilde{a}_{ik}=\sum_{j=1}^{N}a_{ij}a_{jk}\tag{9}$$

$$\tilde{l}_{ik}=\begin{cases}\sum_{k=1,l\neq i}^{N}\tilde{a}_{ik}, i\neq k\\ -\tilde{a}_{ik}, \quad i=k\end{cases}\tag{10}$$

G is a balanced digraph, and we can easily get that $\tilde{G}$ is a balanced digraph. For a balanced digraph with virtual topology, the whole systems can be denoted: $\overline{G}=G+\tilde{G}$. And the Laplacian matrix of the whole systems can be written like that: $\overline{L}_g=L_g+\tilde{L}_g$. Let $u=(u_1,u_2,\dots,u_N)$, $x=(x_1{}^T,x_2{}^T,\dots,x_N{}^T)$, $z=(z_1{}^T,z_2{}^T,\dots,z_N{}^T)$, then we can obtain:

$$\begin{pmatrix}\dot{x}\\ \dot{z}\end{pmatrix}=\begin{pmatrix}I_N\otimes A & I_N\otimes BK_1-\overline{L}_g\otimes BK_2\\ -I_N\otimes LC & I_N\otimes(A+LC)+I_N\otimes BK_1-\overline{L}_g\otimes BK_2\end{pmatrix}\begin{pmatrix}x\\ z\end{pmatrix}\tag{11}$$

Let

$$P=\begin{pmatrix}I_N\otimes A & I_N\otimes BK_1-\overline{L}_g\otimes BK_2\\ -I_N\otimes LC & I_N\otimes(A+LC)+I_N\otimes BK_1-\overline{L}_g\otimes BK_2\end{pmatrix}\tag{12}$$