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Covering points with minimum/maximum area orthogonally convex polygons

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ABSTRACT

In this paper, we address the problem of covering a given set of points on the plane with minimum and/or maximum area orthogonally convex polygons. It is known that the number of possible orthogonally convex polygon covers can be exponential in the number of input points. We propose, for the first time, an $O(n^2)$ algorithm to construct either the maximum or the minimum area orthogonally convex polygon if it exists, else report the non-existence in $O(n \log n)$.

1. Introduction

Reconstruction of shapes from limited stored or discovered geometric and topological information is a frequently visited problem in computational geometry. For example, Jackson and Wismath [1] use "stab visibilities" for reconstructing an orthogonal polygon. Usually, coordinates of the vertices (corners) of an object are used for reconstructing it. O'Rourke [2] provides an $O(n \log n)$ time algorithm for constructing a unique orthogonal polygon, if it exists, from a given vertex set. Löffler and Mumford [3] show that there is only one orientation for which a provided vertex set can be used to reconstruct a rectilinear graph. Rappaport [4] shows that if straight angles are allowed, the problem becomes NP-hard. A similar problem is studied by Biedl and Genç in [5] for reconstructing orthogonally convex polyhedra from putative vertex sets.

In this study, we focus on the reconstruction of polygons when the given points are confined to the interior of the edges. An edge *covers* a point, if the point can be rewritten as a convex combination of the two endpoints of the edge. Similarly, a polygon is said to cover a point set on the plane if each polygon edge covers exactly one point and each point is covered by exactly one polygon edge. The points are therefore required to be in the interior of the polygon edges. An *orthogonal polygon* is a polygon whose edges are orthogonal, namely, horizontal or vertical. The problem of deciding whether a given point set on the plane can be covered by an orthogonal polygon when the orientations of the edges are also specified in advance as part of the input has been shown to be NP-complete by Evrendilek et al. in [6]. In case the orientations are not dictated, the problem remains NP-complete as shown in [7].

The interior of an *x-monotone polygon* intersects every vertical line in at most one line segment. Biedl et al. in [8] studied reconstructing polygons from scanner data such that the provided points are in the interior of the polygon edges, and

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each edge is allowed to cover multiple points. One of their results was that there exists a unique orthogonal x-monotone polygon that can be reconstructed from a given point set in $O(n \log n)$ time if the corresponding edge orientations are known.

An orthogonal polygon is *orthogonally convex* if its intersection with any orthogonal line is either empty or a single line segment. When the covering polygon is restricted to be orthogonally convex, Genç et al. determine in [9] if a given set of points on the plane can be covered by a polygon of this class in $O(n \log n)$ time. It is also shown in [9] that the number of possible orthogonally convex polygons that can cover a given point set can be exponential in the number of the provided points.

In this paper, we give an $O(n^2)$ algorithm to construct either the maximum or the minimum area orthogonally convex polygon which covers a given set of n points, should the provided point set admit one or more (possibly exponentially many) coverings.

There are a variety of domains in which the reconstruction of orthogonally convex polygons with minimum and/or maximum area may find application. It can be employed, for instance, in constructing 2D floor layouts with area constraints, and also as a finer grain substitute for bounding boxes of graphical objects as well as being theoretically interesting in its own right. As stated in [9], orthogonal convex hulls, studied in [10], may be disconnected or degenerate. The orthogonally convex cover, however, is a balanced representation and may be useful where the bounding box is too simple and the orthogonal convex hull is disconnected. In [8], the operation of range scanners situated in a room is described. Such scanners provide data with points in the interior of the polygon edges corresponding to the walls of the room. If the shape of a room is already known to be an orthogonally convex polygon, the construction of its 2D floor layout can be automated.

The rest of the paper is organized as follows: In Section 2, we provide the necessary formal background. Then, in Section 3, the problem of finding the *maximum or minimum area orthogonally convex polygon cover* of a given set of points is defined formally along with the general anatomy of this type of polygons. Subsequently in Section 4, an efficient algorithm is described, and finally, the last section presents concluding remarks.

2. Definitions and background

All the definitions in this section are given for 2D Euclidean Space.

A *polygonal chain* is a connected and ordered set of line segments, such that only consecutive segments intersect and they intersect only at their end points. Likewise, a *closed polygonal chain* is a polygonal chain whose endpoints coincide. A *polygon* is a region on the plane enclosed by a closed polygonal chain. The *edges* of a polygon are the maximal line segments on the boundary of the polygon. The *vertices* of a polygon are the intersection points of its edges. A line segment is *orthogonal* if it is parallel to one of the coordinate axes. An *orthogonal polygon* is a polygon whose edges are orthogonal. An orthogonal polygon is *orthogonally convex*, if its intersection with any orthogonal line is either empty or a single line segment.

An orthogonal line segment is *horizontal* (*resp. vertical*) if it is parallel to the x-axis (resp. y-axis). A line segment *covers* a point, if the point is on the line segment.

Given two points p and q, we write $p \geq_x q$ if the x-coordinate of p is greater than or equal to the x-coordinate of q. $p \geq_y q$ is defined analogously. We require strict inequality for $>_x$ and $>_y$ respectively.

A polygonal chain is x^+-*monotone* if for any two consecutive vertices v_i and v_{i+1}, $v_{i+1} \geq_x v_i$, and it is x^--*monotone* if for any two consecutive vertices v_i and v_{i+1}, $v_i \geq_x v_{i+1}$. We similarly define y^+-*monotone* and y^--*monotone*. A polygonal chain is *xy-monotone* if it is monotone in both orthogonal directions.

3. Problem definition

We can now formally define the problem of finding the *maximum area orthogonally convex polygon cover* of a given set of points. The definition of the problem of finding the minimum area orthogonally convex polygon cover can be obtained by replacing the "maximums" with "minimums".

Definition 1. Given a set P of points in 2D, the problem of finding the maximum area orthogonally convex polygon cover is to construct an orthogonally convex polygon of maximum area in such a way that each edge of the polygon covers exactly a single point, and each point is covered by exactly a single edge of the polygon, if any such polygon exists.

This definition implies that no point in P is allowed to be on a vertex of the orthogonally convex polygon cover as otherwise it would have been covered by the two edges it is on.

Fig. 1 illustrates two different orthogonally convex polygons that both cover the same input point set. For the sake of completeness, and for a better understanding of the solution to this problem, we will introduce the terminology and outline the computations involving the construction of orthogonally convex polygons covering a provided point set in the next subsection.