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ORIGINAL ARTICLE

Distance-based Information Granularity and Hierarchical Structure for an Intuitionistic Fuzzy Granular Space

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Abstract Information granularity and hierarchical structures in granular computing are the two main aspects for investigating the uncertainty and structure of all types of granular spaces. This study presents a distance-based information granularity for IF and multi-granulation IF granular spaces and use it to construct a novel hierarchical structure on such spaces. First, we propose a distance and a relative distance between two IF granular structures to differentiate them and use these two distances to generalize the axiomatic approach of fuzzy information granularity to the IF context. Second, we construct a hierarchical structure based on the relative distance between two IF granular structures to organize the IF granular space hierarchically. Third, we provide the multi-granulation IF granular space and study its relative-distance based axiomatic approach of IF information granularity. Fourth, we propose a relative-distance-based hierarchical structure on multi-granulation IF granular space.

Keywords Granular computing · Distance · Intuitionistic fuzzy information granularity · Hierarchical structure · Multi-granulation

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1. Introduction

Granular computing (GrC), as initiated by Zadeh [1], is an important issue in intelligence information processing [2-4]. Three main research tasks generally occur in GrC, namely, (1) information granulation: how to granulate all the objects in a universe into an information granular structure; (2) information granularity: how to measure the granulation degree of an information granular structure; and (3) hierarchical structure: how to organize all information granular structures in a granular space.

The information granulation of objects in a universe is due to a binary relation, which may be an equivalent relation, a similarity relation, a tolerance relation, a dominance relation, a covering relation, a neighborhood relation, or a fuzzy relation [5-14]. After information granulation, all constructed information granules can lead to a collection of granules called a granular structure, and the collection of all information granules is called a granular space.

After all granular structures are constructed in a granular space, an important concept in GrC is the uncertainty measure of a granular structure, which is a measure of uncertainty about the actual structure of the granular structure. Although all types of uncertainty measures, such as information granularity, information entropy, rough entropy, and information Shannon entropy are proposed [8,15,16], we can call them by a joint name, information granularity. It generally represents the discernibility of an information granule in a granular structure; a small information granularity of a granular structure indicates its strong discernibility. Hence calculating the information granularity of a granular structure is an important issue according to various views and targets [17-22].

Another important concept in GrC is the hierarchical structure of the granular space, which is utilized to analyze the relationships among all granular structures in the granular space. For example, Yao [23] suggested the use of hierarchical granulations to study stratified rough set approximations, Huang et al. proposed the hierarchical structures of a covering-based granulation space in [24], and Yang et al. [25] proposed three hierarchical structures of a multi-granulation granular space. On the basis of knowledge distance, Qian et al. [26] proposed a unified framework for set-based GrC called a lattice model, and She et al. [27] investigated the topological and lattice structures of multi-granulation granular space.

Intuitionistic fuzzy (IF) sets were first formulated by Atanassov [28, 29]. An IF set is a generalization of fuzzy sets. In fuzzy sets, the membership value, $\mu(x)$, of x in the universe of discourse U is only a single real number in [0, 1], and the non-membership of x is taken as $1-\mu(x)$. However, for IF sets, the membership value $\mu(x)$ and the non-membership value $\gamma(x)$ should be considered to describe any x in U such that the sum of membership and non-membership is less than or equal to 1. Thus, an IF set is expressed with an ordered pair of real numbers $<\mu(x), \gamma(x)>$, and $1-\mu(x)-\gamma(x)$ is called the degree of hesitancy. IF set theory was recently applied successfully in decision analysis and pattern recognition [30-34]. Although an IF