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ORIGINAL ARTICLE

On Admissible Total Orders for Interval-valued Intuitionistic Fuzzy Membership Degrees



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Abstract The linearity contained in the natural order of unit interval $[0, 1]$ plays an important role in many concepts and applications of fuzzy theory. Besides, it is very important in concepts like ordered weighted aggregation operators (OWA) and fuzzy decision making. However, this linearity is not inherited by most of fuzzy logics which extend the standard one. To recover the linearity for such frameworks it is required that its related partial order to be extended give rise to the notion of admissible order. In this paper, we study some admissible orders for the framework of interval-valued intuitionistic fuzzy logic.

Keywords Interval-valued intuitionistic fuzzy sets · Score function · Accuracy function · Admissible total orders

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1. Introduction

Since Zadeh's seminal paper [28] several extensions of fuzzy set theory have been proposed [5]. Among them we stress the interval-valued fuzzy set theory [10, 29]

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and the intuitionistic fuzzy set theory¹ [1, 3]. The first captures the intrinsic difficulty to precisely determine the membership degree of an object with respect to some linguistic term; in this case, instead of an exact value, an expert provides an interval which expresses the intrinsic imprecision in his/her degree of assignment [4]. The second adds an extra degree (non-membership degree) to the usual fuzzy sets in order to model the hesitation and uncertainty about the assigned degree of membership. In fuzzy set theory, the non-membership degree is by default the complement of the membership degree, $\mu_A(x)$, i.e., $1 - \mu_A(x)$. Both extensions were combined by Atanassov et al. [2], in this case the underlying imprecision during the process of assignment of membership and non-membership degrees is represented by a pair of intervals. This extension is known as interval-valued intuitionistic fuzzy sets (IVIFSs). There are several applications of IVIFSs as well as some extensions of usual fuzzy concepts to this setting [12, 17, 22].

One of the main advantages of the usual degrees in $[0, 1]$ is that they are linearly ordered, i.e., every pair of such degrees are comparable. This property is fundamental in applications like decision making, where the maximum between any two results is required. Therefore it is interesting that linearity can be extended to the set of membership degrees of IVIFS — abbreviated here by IVIFMD. This is done, here, in such a way that the diagonal set of IVIFMD is order-isomorphic to $[0, 1]$.

Xu and Yager [26] proposed a total order for intuitionistic fuzzy membership degrees based on the notion of score and accuracy functions [11, 15]. The resulting order is admissible, i.e., it extends the natural partial order of intuitionistic fuzzy degrees [8]. The score and accuracy functions for IVIFMD can be extended in several ways, e.g. see [16, 20, 21, 27]; here we propose a new accuracy function for IVIFMD. The notion of admissible total order [8, 9] can be naturally generalized for any poset, in this paper we consider admissible total orders for IVIFMD with their usual partial order. Here we prove that, when we consider several extensions of score and accuracy functions for IVIFMD, the natural extensions of such linear order are not compatible with the natural one, i.e., it does not induce another linear order. Nevertheless, based on the proposal of Xu and Yager [26] we provide a new linear order which is very similar (but not equal) to the admissible total order introduced in [24] which extends the Xu and Yager order for IVIFMD. In addition, we also introduce a family of total orders for IVIFMD based on arbitrary total orders for interval-valued fuzzy degrees and analyze their admissibility.

This paper is organized as follows: Section 2 contains the notions of intuitionistic fuzzy sets, interval-valued fuzzy sets (IVFSs) and some related notions such as score and accuracy indexes. Section 3 presents the notion of IVIFS as well as some notions and notations, particularly some extensions of score and accuracy indexes for IVIFMD are analyzed. Section 4 provides two total orders and a parameterized family of total orders for IVIFMD, and their properties are analyzed. Finally, some conclusions and future research are pointed out in Section 5.

¹ This notion is not related with the fuzzy extension of the intuitionistic logic of Brouwer [18] proposed by Takeuti and Titani in [23]. In order to avoid this confusion, some authors have used the name “Atanassov intuitionistic fuzzy logic” but we opted to omit the name of Atanassov by considering that the readers are clear that here it is being used the approach of Atanassov.

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