Fuzzy Inf. Eng. (2016) 8: 183–198

Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Information and Engineering
http://www.elsevier.com/locate/fiae

ORIGINAL ARTICLE

Robustness of Fuzzy Reasoning Based on Schweizer-Sklar Interval-valued t-Norms

Min-Xia Luo · Ze Cheng

Received: 6 August 2015/ Revised: 5 April 2016/
Accepted: 20 April 2016/

Abstract In this paper, we focus on the parametric triple I algorithms by the combination of Schweizer-Sklar interval-valued operators and triple I principles for fuzzy reasoning. Firstly, we give the interval-valued triple I solutions based on Schweizer-Sklar interval-valued operators. Then, we investigate the sensitivity of Schweizer-Sklar interval-valued fuzzy connectives. Finally, we study the robustness of the triple I algorithms based on Schweizer-Sklar interval-valued t-norms ($m \in (0, \infty)$). It shows that the quality of interval-valued fuzzy reasoning algorithms depends on the selection of interval-valued fuzzy connectives.

Keywords Schweizer-Sklar interval-valued operator · Interval-valued fuzzy connective · Interval-valued fuzzy inference · Robustness

1. Introduction

It is well known that the most fundamental forms of fuzzy reasoning are fuzzy modus ponens (FMP) and fuzzy modus tollens (FMT), which can be respectively expressed as follows [1, 2]:

Given the input "x is A^* " and fuzzy rule " if x is A then y is B ", try to deduce a reasonable output "y is B^* ", FMP.

Given the input "y is B^* " and fuzzy rule " if x is A then y is B ", try to deduce a reasonable output "x is A^* ", FMT.

Min-Xia Luo (✉) · Ze Cheng
Department of Mathematics, China Jiliang University, Hangzhou 310018, P.R. China
email: minxialuo@163.com
Peer review under responsibility of Fuzzy Information and Engineering Branch of the Operations Research Society of China.

http://dx.doi.org/10.1016/j.fiae.2016.06.004

In the above models, A and A^* belong to fuzzy sets $\mathcal{F}(X)$ in the non-empty set X, B and B^* belong to fuzzy sets $\mathcal{F}(Y)$ in the non-empty set Y.

To solve fuzzy reasoning FMP problem, the most basic method is Zadeh's compositional rule of inference (CRI for short) [2], it has been applied successfully in many fields. But the CRI method has some disadvantages [3-5], for example, the composition operation in the CRI method is short of clear logic meaning, and the CRI method is not reductive. Based on these disadvantages, a new method was proposed by Wang [3], called the full implication triple I method of fuzzy reasoning. This method may bring fuzzy reasoning within the framework of logical semantic implication [1], and it can be considered as a reasonable complement for the CRI method. Consequently, a considerable number of studies on the triple I method have been reported in recent years. Based on regular implications and normal implications, the unified triple I algorithms have been established by Wang and Fu [5]. For all residuated implications induced by left continuous t-norms, unified full implication triple I algorithms and unified full implication α-triple I algorithms of fuzzy reasoning were constructed by Pei [7]. The parametric triple I algorithms by the combination of Schweizer-Sklar operators and triple I principles for fuzzy reasoning were investigated by Luo and Yao [8].

Since type-2 fuzzy set introduced by Zadeh [9] can provide us with more design degrees of freedom, type-2 fuzzy reasoning has been generally acknowledged as being advantageous and potential in uncertainty modeling [10]. As a special type-2 fuzzy set, interval-valued fuzzy set (of which traditional $[0,1]$-valued membership degrees are replaced by intervals in $[0,1]$) can not only effectively reduce the loss of fuzzy information but also reflect the vagueness and uncertainty in information processing. Furthermore, interval-valued fuzzy set is easier to handle than type-2 fuzzy set in practical applications. A constructive method to the definition of interval-valued fuzzy implication operators was introduced by C. Alcalde [11].

Interval-valued fuzzy reasoning has become very popular in various fields such as network, control systems and so on. Therefore, it becomes a favorite topic to study interval-valued fuzzy reasoning. Particularly, robustness analysis attracts more and more attention in these years. Compositional rule of inference and the interval-valued fuzzy set was combined (ICRI for short), and the robustness of interval-valued fuzzy reasoning based on ICRI was discussed by Li et al. [12]. Full implication triple I algorithms was extended to the case of interval-valued fuzzy set, and the robustness of full implication algorithms based on interval-valued fuzzy inference was investigated by Luo [13].

The paper focuses on the parametric triple I algorithms by the combination of Schweizer-Sklar interval-valued operators and triple I principles for fuzzy reasoning. In Section 2, we introduce these definitions about Schweizer-Sklar interval-valued t-norms, interval-valued residuated implication operators and interval-valued t-conorms. The interval-valued triple I solutions based on Schweizer-Sklar interval-valued operators are given. In Sections 3, the perturbation of Schweizer-Sklar interval-valued fuzzy connectives are investigated. In Sections 4, robustness of the triple I algorithms are proved based on Schweizer-Sklar interval-valued t-norms. Finally, we conclude a short summary in Section 5.