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Uniform strategies, rational relations and jumping automata

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A B S T R A C T

A general concept of uniform strategies has recently been proposed as a relevant notion in game theory for computer science, which subsumes various notions from the literature. It relies on properties involving sets of plays in two-player turn-based arenas equipped with arbitrary binary relations between plays; these properties are expressed in a language based on CTL* with a quantifier over related plays. There are two semantics for our quantifier, a *strict* one and a *full* one, that we study separately. Regarding the strict semantics, the existence of a uniform strategy is undecidable for *rational* binary relations, but introducing *jumping tree automata* and restricting attention to *recognizable* relations allows us to establish a 2-EXPTIME-complete complexity – and still capture a class of two-player imperfect-information games with epistemic temporal objectives. Regarding the full semantics, relying on *information set automata* we establish that the existence of a uniform strategy is decidable for rational relations and we provide a nonelementary synthesis procedure. We also exhibit an essentially optimal subclass of rational relations for which the problem becomes 2-EXPTIME-complete. Considering rich classes of relations makes the theory of uniform strategies powerful: it directly entails various results in logics of knowledge and time, some of them already known, and others new.

1. Introduction

Infinite-duration game models have been intensively studied for their applications in computer science [2] and logic [19]. First, infinite-duration games provide a natural abstraction of computing systems' non-terminating interaction [1] – think of a communication protocol between a printer and its users, or control systems. Second, infinite-duration games naturally occur as a tool to handle logical systems for the specification of non-terminating behaviours, such as the propositional μ-calculus [16], leading to a powerful theory of automata, logics and infinite games [19] and to the development of algorithms for the automatic verification ("model-checking") and synthesis of hardware and software systems. In all cases, solving games aims at computing a strategy (of some distinguished player) whose outcomes fulfil ω-regular conditions meant to describe some desirable property.

In essence, ω-regular conditions are evaluated on individual plays, independently of other plays that result from the strategy. However, turning to imperfect-information games raises the need to deal with *sets* of plays, as the strategic decision has to be the same in indistinguishable situations [39]. This typical property of strategies in imperfect-information games is in general dealt with aside from the ω-regular winning conditions. However, this splitting is a real issue when considering properties of strategies that mix, e.g. knowledge and time.

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In an attempt to study this problem in depth, Maubert and Pinchinat [29] introduced a general notion of *uniform strategies* and showed that it captures a variety of settings from the literature. Uniformity properties of strategies are expressed in a logic that combines standard temporal modalities with two new quantifiers, $\boxed{\rightsquigarrow}$ and $\boxed{\boxed{\rightsquigarrow}}$, that universally quantify over "related" plays according to a binary relation between plays. The difference between the two quantifiers is their range. While the *strict* quantifier $\boxed{\rightsquigarrow}$ only quantifies over related plays that follow the strategy, the *full* quantifier $\boxed{\boxed{\rightsquigarrow}}$ ranges over all related plays in the arena.

These quantifiers generalize the knowledge operator K of epistemic temporal logics: classically, the semantics of K is a universal quantification over histories related to the actual one by some observational equivalence relation that captures the capabilities of the agent – perfect/imperfect recall, synchronous/asynchronous, ... [20]. In contrast, the setting of Maubert and Pinchinat [29] allows arbitrary binary relations as long as they are *rational*, i.e. recognized by *finite state transducers* [4,14]. Noticeably, most equivalence relations used in epistemic temporal logics are recognized by very simple transducers. Additionally, rational relations need not be equivalences, and can capture relations used in belief revision, and for modelling plausibility, with K45 or KD45 axiomatization [17].

In this work we extend the setting of Maubert and Pinchinat [29] by founding our logical language $\mathcal{L}_{\rightsquigarrow}$ on the full branching time logic CTL* instead of LTL, allowing us to capture e.g. games with CTL* winning conditions and module-checking [22]. Constraining the uniformity properties to use only either the strict quantifier or the full quantifier yields the notions of *strictly-uniform strategies* and *fully-uniform strategies*, which we study separately as they require different techniques.

Relying on Maubert and Pinchinat [28], we first establish the undecidability of the *strictly-uniform strategy problem* (i.e. the existence of a strictly-uniform strategy) when the whole class of rational relations is considered. More precisely, we show that this undecidability result holds even if we restrict attention to the subclass of regular[1] equivalence relations. To try and better understand the difficulty of this problem, we propose an automata-based approach inspired by Vardi [41] for solving LTL games. We introduce and study *jumping tree automata (JTA)*, a class of tree automata which generalizes standard alternating tree automata. JTA are equipped with a binary relation between branches of trees and, in addition to the usual behaviour of alternating automata, they allow for jumps between related nodes of the input tree. Intuitively, the jumps of JTA "implement" the meaning of the $\boxed{\rightsquigarrow}$ quantifier in $\mathcal{L}_{\rightsquigarrow}$. We show that JTA capture the full logic $\mathcal{L}_{\rightsquigarrow}$, and that from a uniformity property we can build a JTA that accepts the tree unfoldings of strictly-uniform strategies.

Although the emptiness problem for JTA is unsurprisingly undecidable when considering arbitrary rational relations over branches of trees, we identify a decidable case when the class of binary relations between branches is confined to the well-known family of *recognizable* relations; basically, such relations only challenge a bounded amount of information in each branch. Decidability of JTA emptiness in this case is shown by an effective transformation of JTA with recognizable relations into equivalent two-way tree automata. The emptiness problem for JTA with recognizable relations is then Exptime-complete, and the strictly-uniform strategy problem for this class of relations is 2-Exptime-complete.

Concerning fully-uniform strategies, deciding their existence (the *fully-uniform strategy problem*) has been investigated by Maubert and Pinchinat [29] for the case of linear time modalities. The problem is in k-Exptime for logical specifications that involve up to k nested $\boxed{\boxed{\rightsquigarrow}}$ quantifiers – 2-Exptime if $k \leq 2$. We prove that these complexities still hold when the full branching time logic CTL* is allowed, and we establish the matching lower bounds. We also introduce information set automata as a tool to compute a generalized notion of information sets for rational relations. Information set automata offer a modular proof of our decision procedure, and they enable us to identify a rich subclass of rational relations (K45NM) that still contains relations considered in epistemic temporal logics and games with imperfect information, and for which the fully-uniform strategy problem is 2-Exptime-complete.

At last, we generalize these results to the case of multiple relations $\rightsquigarrow_i$ with corresponding quantifiers $\boxed{\rightsquigarrow}_i$ and $\boxed{\boxed{\rightsquigarrow}}_i$, and we describe how several problems from the literature fit in this generalized setting. We show that several known results find a unified proof in our work, and we fill some gaps; we finally take inspiration from other related results to discuss future work.

The rest of the paper is organized as follows. In Section 2, we recall some notions concerning words, trees, game arenas and rational relations. We present in Section 3 the language $\mathcal{L}_{\rightsquigarrow}$ to specify uniform strategies, and we define them, before giving in Section 4 two examples of problems that uniform strategies naturally capture. In Section 5 we study the strictly-uniform strategy problem: we prove that it is undecidable in general and, resorting to jumping tree automata, we establish its 2-Exptime-completeness in the case of recognizable relations. Section 6 starts with the statement of our nonelementary result for the fully-uniform strategy problem and the elementary case of K45NM relations, it continues with the exposition of information set automata, which we use to establish our upper bounds, and the section ends with the proofs for the lower bounds. We extend our results to the case of multiple relations and combinations of strict and full quantifiers in Section 7, in which we also state several corollaries and discuss related work. We conclude and give perspectives in Section 8.

Because of space limitation, some proofs are omitted, and others are just sketched. However, all details can be found in Maubert [27].

[1] Captured by synchronous transducers.