Information and Computation 242 (2015) 183–226

Contents lists available at ScienceDirect

Information and Computation

www.elsevier.com/locate/yinco

Model checking single agent behaviours by fluid approximation

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ARTICLE INFO

Article history:
Received 14 January 2013
Received in revised form 12 May 2014
Available online 6 March 2015

Keywords:
Stochastic model checking
Fluid approximation
Mean field approximation
Reachability probability
Time-inhomogeneous Continuous Time Markov Chains

ABSTRACT

In this paper we investigate a potential use of fluid approximation techniques in the context of stochastic model checking of CSL formulae. We focus on properties describing the behaviour of a single agent in a (large) population of agents, exploiting a limit result known also as fast simulation. In particular, we will approximate the behaviour of a single agent with a time-inhomogeneous CTMC, which depends on the environment and on the other agents only through the solution of the fluid differential equation, and model check this process. We will prove the asymptotic correctness of our approach in terms of satisfiability of CSL formulae. We will also present a procedure to model check time-inhomogeneous CTMC against CSL formulae.

1. Introduction

In recent years, there has been a growing interest in fluid approximation techniques in the formal methods community [1–6]. These techniques, also known as mean field approximation, are useful for analysing quantitative models of systems based on continuous time Markov Chains (CTMC), possibly described in process algebraic terms. They work by approximating the discrete state space of the CTMC by a continuous one, and by approximating the stochastic dynamics of the process with a deterministic one, expressed by means of a set of differential equations. The asymptotic correctness of this approach is guaranteed by limit theorems [7–9], showing the convergence of the CTMC to the fluid ODE for systems of increasing size.

The notion of size can be different from domain to domain, yet in models of interacting agents usually considered in computer science, the size has the standard meaning of number of individuals in a population. All these fluid approaches, in particular, require a shift from an agent-based description to a population-based one, in which the system is represented by variables counting the number of agents in each possible state and so individual behaviours are abstracted. In fact, in large systems, the individual choices of single agents have a small impact, hence the whole system tends to evolve according to the average behaviour of agents. Therefore, the deterministic description of the fluid approximation is mainly related to the average behaviour of the model, and information about statistical properties is generally lost, although it can be partially recovered by introducing fluid equations of higher order moments of the stochastic process (moment closure techniques [10–12]).

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http://dx.doi.org/10.1016/j.ic.2015.03.002

When kept discrete, quantitative systems like those described by process algebras can be analysed using quantitative model checking. These techniques have a long tradition in computer science and are powerful ways of querying a model and extracting information about its behaviour. As far as stochastic model checking is considered, there are some established approaches based mainly on checking Continuous Stochastic Logic (CSL) formulae [13–15], which led to widely used software tools [16]. All these methods, however, suffer (in a more or less relevant way) from the curse of state space explosion, which severely hampers their practical applicability. In order to mitigate these combinatorial barriers, multiple techniques have been developed, many of them based on some notion of abstraction or approximation of the original process [17,18].

In this paper, we will precisely target this problem, investigating the extent to which fluid approximation techniques can be used to speed up the model checking of CTMC. We will focus on a restricted subset of system properties: We will consider population models in which many agents interact, and then focus on the behaviour of individual agents. In fact, even if large systems behave almost deterministically, the evolution of a single agent in a large population is always stochastic. Single agent properties are interesting in many application domains. For instance, in performance models of computer networks, like client-server interactions, one is often interested in the behaviour and quality-of-service metrics of a single client (or a single server), such as the waiting time of the client or the probability of a time-out.

Single agent properties may also be interesting in other contexts. In ecological models, one may be interested in the chances of survival or reproduction of an animal, or in its foraging patterns [19]. In biochemistry, there is some interest in the stochastic properties of single molecules in a mixture (single molecule enzyme kinetics [20,21]). Other examples may include the time to reach a certain location in a traffic model of a city, or the chances to escape successfully from a building in case of emergency egress [22].

The use of fluid approximation in this restricted context is made possible by a corollary of the fluid convergence theorems, known by the name of *fast simulation* [23,9], which provides a characterization of the behaviour of a single agent in terms of the solution of the fluid equation: the agent senses the rest of the population only through its "average" evolution, as given by the fluid equation. This characterization can be proved to be asymptotically correct.

The main idea of this paper is simply to use the CTMC for a single agent obtained from the fluid approximation instead of the full model with N interacting agents. In fact, extracting metrics from the description of the global system can be extremely expensive from a computational point of view. Fast simulation, instead, allows us to abstract the system and study the evolution of a single agent (or of a subset of agents) by decoupling its evolution from the evolution of its environment. This has the effect of drastically reducing the dimensionality of the state space by several orders of magnitude.

Of course, in applying the mean field limit, we are introducing an error which is difficult to control (there are error bounds but they depend on the final time and they are very loose [9]). However, this error decreases as the populations increase, and it is usually acceptable in practice, especially for systems with a large pool of agents, as certified by the widespread use of fluid approximation [24]. We stress that these are precisely the cases in which current tools suffer severely from state space explosion, and that can benefit most from a fluid approximation. However, we will see in the following that in many cases the quality of the approximation is good also for small populations.

In the rest of the paper, we will basically focus on how to analyse single agent properties of three kinds:

- Next-state probabilities, i.e. the probability of jumping into a specific set of states, at a specific time.
- Reachability properties, i.e. the probability of reaching a set of states G, while avoiding unsafe states U.
- Branching temporal logic properties within a bounded amount of time, i.e. verifying time-bounded CSL formulae.

A central feature of the abstraction based on fluid approximation is that the limit of the model of a single agent has rates depending on time, via the solution of the fluid ODE. Hence, the limit models are time-inhomogeneous CTMC (ICTMC). This introduces some additional complexity in the approach, as model checking of an ICTMC is far more difficult than in the standard time-homogeneous case. To the best of the authors' knowledge, in fact, there is no known algorithm to solve this problem in general, although related work is presented in Section 2. We will discuss a general method in Section 5, based on the solution of variants of the Kolmogorov equations, which is expected to work for small state spaces and the controlled dynamics of the fluid approximation. The main difficulty with CSL model checking of ICTMC is that the truth of a formula can depend on the time at which the formula is evaluated. Hence, we need to impose some regularity on the dependency of rates on time to control the complexity of time-dependent truth. We will see that the requirement, piecewise analyticity of rate functions, is intimately connected not only with the decidability of the model checking for ICTMC, but also with the lifting of convergence results from CTMC to truth values of CSL formulae (Theorems 5.1 and 6.1).

Summarising, the main contributions of the paper are the following:

- Methodologically, we advocate the use of fluid approximation to efficiently verify properties of individual agents in large population models, dubbing this approach *fluid model checking*.
- We present a novel model checking algorithm for time-bounded CSL properties on time-inhomogeneous CTMCs.
- We prove that asymptotic correctness of fluid model checking for time-bounded CSL formulae.

The structure of the paper reflects the three main contributions listed above. We start by discussing related work in Section 2, and by introducing preliminary notions, fixing the class of models considered (Section 3.1), presenting fluid limit and fast simulation theorems (Sections 3.2 and 3.3), and introducing Continuous Stochastic Logic (Section 3.4) and