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The g-good-neighbor conditional diagnosability of n-dimensional hypercubes under the MM* model ☆

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ABSTRACT

Diagnosability of a multiprocessor system is one important study topic. In 2012, Peng et al. proposed a new measure for fault diagnosis of the system, which is called g-good-neighbor conditional diagnosability that restrains every fault-free node containing at least g fault-free neighbors. As a famous topology structure of interconnection networks, the n-dimensional hypercube has many good properties. In this paper, we give the g-good-neighbor conditional diagnosability of the n-dimensional hypercube under the MM* model.

1. Introduction

As a famous topology structure of interconnection networks, the n-dimensional hypercube Q_n has many good properties, such as connectivity, symmetry, short communication distance, and high fault tolerance of network. Q_n is the first network structure which is taken into practice on IBM machine diagram. With the development of interconnection network topologies, there are many deformation networks which are based on the hypercube, such as the Crossed cube, the Twisted cube and the Möbius cube. As the size of multiprocessor systems increases continuously, the possibility of the systems existing more fault processors increases greatly. In order to maintain the reliability of the systems, whenever a processor is found faulty, it should be replaced by a fault-free processor. Several diagnosis models were proposed to identify the faulty processors. One major approach is the Preparata, Metze, and Chien's (PMC) diagnosis model introduced by Preparata et al. [7]. The diagnosis of the system is achieved through two linked processors testing each other. Another important model, namely the comparison diagnosis model (MM model), was proposed by Maeng and Malek [5]. In the MM model, to diagnose a system, a node sends the same task to two of its neighbors, and then compares their responses. A system is t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. Diagnosability of a system is an important parameter for system diagnosis, which is the maximum value of t such that the system is t-diagnosable. For a t-diagnosable system, Dahbura and Masson [4] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors.

An n-dimensional hypercube Q_n $(n \geq 1)$ is the graph whose vertex set is the set of all n-tuples of 0s and 1s,

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where two n-tuples are adjacent if they differ in precisely one coordinate. Q_n has $\binom{2^n}{n}$ vertex subsets of size n, among which there are only 2^n vertex subsets which contain all the neighbors of some vertex. Since the ratio $2^n/\binom{2^n}{n}$ is very small for large n, the probability of a fault set with size n containing all the neighbors of any vertex is very low. So Lai et al. [2] introduced the restricted diagnosability of multiprocessor systems called conditional diagnosability. They consider the situation that any fault set cannot contain all the neighbors of any vertex in a system. Inspired by this concept, Peng et al. [6] proposed a new measure for fault diagnosis of the system, namely, g-good-neighbor conditional diagnosability, which requires that every fault-free node contains at least g fault-free neighbors. In [6], they studied the g-good-neighbor conditional diagnosability of the n-dimensional hypercube Q_n under PMC model. Later, Yuan et al. [9] studied that the g-good-neighbor conditional diagnosability of the k-ary n-cube ($k \geq 4$) under the PMC model and MM* model. In this paper, we give that the g-good-neighbor conditional diagnosability of Q_n under the MM* model is $(n-g+1)2^g-1$, where $n \geq 5$ and $0 \leq g \leq n-3$.

2. Terminology and notations

2.1. Terminology

A multiprocessor system is modeled as an undirected simple graph $G=(V,E)$, whose vertices represent processors and edges represent communication links. Given a nonempty vertex subset V' of V, the induced subgraph by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V'. The degree $d_G(v)$ of a vertex v is the number of edges incident with v. We denote by $\delta(G)$ the minimum degrees of vertices of G. For any vertex v, we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor or a neighbor vertex of v for $u \in N_G(v)$. Let $S \subseteq V(G)$. We use $N_G(S)$ to denote the set $\cup_{v\in S} N_G(v)\setminus S$, $C_G(S)$ to denote the set $N_G(S) \cup S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if for any vertex v, $d_G(v)=k$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left. We say that the g-good-neighbor property of H, $P_g(H)$ holds for H if and only if every vertex in H has at least g neighbors in H. Let $G=(V,E)$. A fault set $F \subseteq V$ is called a g-good-neighbor conditional faulty set if $|N(v) \cap (V\setminus F)| \geq g$ for every vertex v in $V\setminus F$. A g-good-neighbor conditional cut of a graph G is a g-good-neighbor conditional faulty set F such that $G-F$ is disconnected. The cardinality of the minimum g-good-neighbor conditional cut is said to be the R_g-connectivity of G, denoted by $\kappa^{(g)}(G)$. Let F_1 and F_2 be two distinct subsets of V, and let the symmetric difference $F_1 \triangle F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. For graph-theoretical terminology and notation not defined here we follow [1].

2.2. The MM* model

In the MM model [5,9], to diagnose a system, a vertex sends the same task to two of its neighbors, and then compares their responses. To be consistent with the MM model, we have the following assumptions.

1. All faults are permanent.

2. A faulty processor produces incorrect outputs for each of its given tasks.

3. The output of a comparison performed by a faulty processor is unreliable.

4. Two faulty processors given the same input and task do not produce the same output.

The comparison scheme of a system G is modeled as a multigraph, denoted by $M(V(G),L)$, where L is the labeled-edge set. A labeled edge $(u,v)_w \in L$ represents a comparison in which two vertices u and v are compared by a vertex w, which implies $uw, vw \in E(G)$. The collection of all comparison results in $M(V(G),L)$ is called the syndrome, denoted by σ^*, of the diagnosis. If the comparison $(u,v)_w$ disagrees, then $\sigma^*((u,v)_w)=1$; otherwise, $\sigma^*((u,v)_w)=0$. Hence, a syndrome is a function from L to $\{0,1\}$. The MM* model is a special case of the MM model. In the MM* model, all comparisons of G are in the comparison scheme of G, i.e., if $uw, vw \in E(G)$, then $(u,v)_w \in L$.

3. Diagnosability of the n-dimensional hypercube under the comparison model

The n-dimensional hypercube Q_n is proposed by Saad and Schultz [8], which is a famous topology structure of interconnection network for multiprocessor systems. Q_n is an undirected n-regular graph containing 2^n vertices, which has the form $u_1u_2\cdots u_n$, where $u_i \in \{0,1\}$ for $1 \leq i \leq n$. Two vertices $u=u_1u_2\cdots u_n$ and $v=v_1v_2\cdots v_n$ are adjacent if and only if there exists exactly a dimension j such that $u_j \neq v_j$ for every $1 \leq j \leq n$, i.e., $\Sigma_{i=1}^{n}|u_i-v_i|=1$. In this paper, we give the g-good-neighbor conditional diagnosability of Q_n under the MM* model.

Before discussing the g-good-neighbor conditional diagnosability of the n-dimensional hypercube under the MM* model, we first give some useful known theorems.

Theorem 3.1. *(See [4,9].) A system G is g-good-neighbor conditional t-diagnosable under the MM* model if and only if for each distinct pair of g-good-neighbor conditional faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$ satisfies one of the following conditions.*

(1). There are two vertices $u, w \in V(G)\setminus(F_1 \cup F_2)$ and there is a vertex $v \in F_1 \triangle F_2$ such that $uw \in E$ and $vw \in E$.

(2). There are two vertices $u, v \in F_1 \setminus F_2$ and there is a vertex $w \in V(G)\setminus(F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.

(3). There are two vertices $u, v \in F_2 \setminus F_1$ and there is a vertex $w \in V(G)\setminus(F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.

Theorem 3.2. *(See [6,8].) Let $n \geq 3$ and $0 \leq g \leq n-2$. Suppose that F is a minimum cardinality cut of Q_n such that $|N_{Q_n}(x) \setminus F| \geq g$ for all $x \in V(Q_n)\setminus F$. Then $|F|=(n-g)2^g$.*