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Perfect quantum teleportation by four-particle cluster state

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A B S T R A C T

Two perfect quantum teleportation schemes for an unknown single-particle pure state and an arbitrary two-particle pure entangled state are proposed by using four-particle cluster state in this paper. As it is shown, the unknown single-particle pure state or the arbitrary two-particle pure entangled state can be teleported perfectly. The successful possibilities are both 100% and quantum channel fidelities are both 1 about these two schemes.

1. Introduction

A modern viewpoint is to regard quantum entangled states as resource for certain communication and computational tasks such as quantum secret sharing [1], quantum secure direct communication [2], quantum private comparison [3], quantum teleportation (i.e. QT) [4–9], and so on. QT is a process to transfer an unknown particle state from a sender Alice to a receiver Bob with the help of some classical information. Bennett et al. [5] firstly presented the theoretical QT protocol for an unknown single-particle state by using maximally entangled two-particle states in 1993. The various QT schemes for an unknown single-particle state [10–15] and an arbitrary two-particle state [9,16] are proposed. In 1998, Karlsson and Bourennane [10] proposed the first controlled teleportation protocol with a three-particle Greenberger–Horne–Zeilinger state. After this, controlled teleportation is developed. S.S. Li et al. [6] proposed controlled QT to teleport a single-particle state and an two-particle entangled state under the control of the supervisor using four-particle cluster state. Y.Y. Nie et al. [7] proposed non-maximally entangled controlled teleportation using four particles cluster states to teleport a single-particle state. Here, we point out that the four-particle cluster state is widely applied since it has many particular characters which are shown in Ref. [17]. The cluster states have two basic properties maximum connectedness and a high persistency of entanglement. It is harder disentangled than GHZ-class states by local measurements. Therefore, many teleportation schemes are proposed based on four-particle cluster state [6–9]. X.W. Wang et al. [8] present a quantum teleportation of an arbitrary two-particle state by using one-dimensional cluster state. C.D. Li et al. [9] proposed a teleportation of two-particle entangled state via cluster state. Recently, many teleportation protocols using the multi-particle cluster state are proposed [18–20]. Transferring a three-particle state by using a five-particle cluster state of QT is proposed by Zhong and Lin [18]. Since an arbitrary single-particle state can be teleported in [5]. It becomes possible to teleport an unknown entangled state from one party to the other party. In our paper, not only an arbitrary single-particle state can be teleported but also an arbitrary two-particle entangled state can be teleported. The motivation or the significance

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of the research is to realize that multi-particle entangled states can be teleported by using multi-particle cluster states.

In this paper, we proposed two new teleportation schemes. QT of an unknown single-particle pure state and an arbitrary two-particle pure entangled state can be realized by using four-particle cluster state. The successful possibilities are 100% and the quantum channel fidelities are both 1 for these two schemes.

The structure of this paper is organized as follows. We present the quantum teleportation of an unknown single-particle pure state in Sect. 2 and for an arbitrary two-particle pure entangled state in Sect. 3 in detail. At last, we analyze the performance of our two schemes in Sect. 4 and make a conclusion in Sect. 5.

2. Quantum teleportation of an unknown single-particle pure state

Suppose Alice has an unknown single-particle pure state, which is

$$|\chi\rangle_a = \alpha|0\rangle_a + \beta|1\rangle_a, \tag{1}$$

where $|\alpha|^2 + |\beta|^2 = 1$. Alice and Bob share a four-particle cluster state

$$|\psi\rangle_{1234} = \frac{1}{2}[|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle]_{1234}. \tag{2}$$

The particles 1, 2, 3 belong to Alice, and particle 4 belongs to Bob. The state of the whole system is

$$\begin{aligned}|\Phi_1\rangle &= |\chi\rangle_a \otimes |\psi\rangle_{1234}\\ &= \frac{1}{2}[|\varphi_1\rangle_{a123}(\alpha|0\rangle_4 + \beta|1\rangle_4)\\ &\quad + |\varphi_2\rangle_{a123}(\alpha|0\rangle_4 - \beta|1\rangle_4)\\ &\quad + |\varphi_3\rangle_{a123}(\alpha|1\rangle_4 + \beta|0\rangle_4)\\ &\quad + |\varphi_4\rangle_{a123}(\alpha|1\rangle_4 - \beta|0\rangle_4)],\end{aligned} \tag{3}$$

where $|\varphi_i\rangle_{a123} (i = 1, 2, 3, 4)$ are mutually orthogonal four-particle states given by

$$\begin{aligned}|\varphi_1\rangle_{a123} &= \frac{1}{2}[|0000\rangle + |0110\rangle + |1001\rangle - |1111\rangle]_{a123},\\ |\varphi_2\rangle_{a123} &= \frac{1}{2}[|0000\rangle + |0110\rangle - |1001\rangle + |1111\rangle]_{a123},\\ |\varphi_3\rangle_{a123} &= \frac{1}{2}[|0001\rangle - |0111\rangle + |1000\rangle + |1110\rangle]_{a123},\\ |\varphi_4\rangle_{a123} &= \frac{1}{2}[|0001\rangle - |0111\rangle - |1000\rangle - |1110\rangle]_{a123}.\end{aligned} \tag{4}$$

Alice measures particles a, 1, 2 and 3 by using measurement basis $|\varphi_i\rangle_{a123} (i = 1, 2, 3, 4)$. The measurement results correspond to two classical bits. Alice transforms the two classical bits to Bob by classical channel. Bob applies appropriate Pauli operations to recover the unknown single-particle pure state. Alice's measurement results and corresponding operations of Bob are listed in **Table 1**.

Table 1
Bob's transformations for recovering the unknown single-particle pure state.

Alice's results	Classical bits	Bob's state	Bob's unitary transformations
$\|\varphi_1\rangle_{a123}$	00	$\alpha\|0\rangle_4 + \beta\|1\rangle_4$	I
$\|\varphi_2\rangle_{a123}$	01	$\alpha\|0\rangle_4 - \beta\|1\rangle_4$	σ_z
$\|\varphi_3\rangle_{a123}$	10	$\alpha\|1\rangle_4 + \beta\|0\rangle_4$	σ_x
$\|\varphi_4\rangle_{a123}$	11	$\alpha\|1\rangle_4 - \beta\|0\rangle_4$	$i\sigma_y$

According to the principle of quantum teleportation, Alice transforms his own four states $\{|\varphi_1\rangle_{a123}, |\varphi_2\rangle_{a123}, |\varphi_3\rangle_{a123}, |\varphi_4\rangle_{a123}\}$ into two classical bits {00, 01, 10, 11} respectively. Alice and Bob both know these transformations. The reason, which the four states are transformed by two classical bits rather than one classical bit or three classical bits or more classical bits, is that the two classical bits are enough to be distinguished.

3. Quantum teleportation of an arbitrary two-particle pure entangled state

Suppose Alice has an arbitrary two-particle pure entangled state as well, which is given by

$$|\theta\rangle_{ab} = \alpha|00\rangle_{ab} + \beta|01\rangle_{ab} + \gamma|10\rangle_{ab} + \delta|11\rangle_{ab}, \tag{5}$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Controlled teleportation of an arbitrary two-particle pure or mixed state by using a five-qubit cluster state was proposed in Ref. [19]. We only consider to teleport pure state in this paper and will discuss how to teleport a mixed state in the later research. The quantum channel is also the four-particle cluster state in Eq. (2). Particles 1, 4 belong to Alice, and particles 2, 3 belong to Bob. The whole system is

$$\begin{aligned}|\Phi_2\rangle &= |\theta\rangle_{ab} \otimes |\psi\rangle_{1234}\\ &= \frac{1}{4}[|\phi_1\rangle_{ab14}(\alpha|00\rangle_{23} + \beta|01\rangle_{23}\\ &\quad + \gamma|10\rangle_{23} + \delta|11\rangle_{23})\\ &\quad + |\phi_2\rangle_{ab14}(\alpha|00\rangle_{23} - \beta|01\rangle_{23}\\ &\quad - \gamma|10\rangle_{23} + \delta|11\rangle_{23})\\ &\quad + |\phi_3\rangle_{ab14}(\alpha|00\rangle_{23} - \beta|01\rangle_{23}\\ &\quad + \gamma|10\rangle_{23} - \delta|11\rangle_{23})\\ &\quad + |\phi_4\rangle_{ab14}(\alpha|00\rangle_{23} + \beta|01\rangle_{23}\\ &\quad - \gamma|10\rangle_{23} - \delta|11\rangle_{23})\\ &\quad + |\phi_5\rangle_{ab14}(\alpha|01\rangle_{23} + \beta|00\rangle_{23}\\ &\quad + \gamma|11\rangle_{23} + \delta|10\rangle_{23})\\ &\quad + |\phi_6\rangle_{ab14}(\alpha|01\rangle_{23} - \beta|00\rangle_{23}\\ &\quad - \gamma|11\rangle_{23} + \delta|10\rangle_{23})\\ &\quad + |\phi_7\rangle_{ab14}(\alpha|01\rangle_{23} - \beta|00\rangle_{23}\\ &\quad + \gamma|11\rangle_{23} - \delta|10\rangle_{23})\\ &\quad + |\phi_8\rangle_{ab14}(\alpha|01\rangle_{23} + \beta|00\rangle_{23}\\ &\quad - \gamma|11\rangle_{23} - \delta|10\rangle_{23})\end{aligned}$$