Journal of Computer and System Sciences 81 (2015) 1311–1332

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

A complete parameterized complexity analysis of bounded planning ☆

Christer Bäckström [a], Peter Jonsson [a], Sebastian Ordyniak [b,*], Stefan Szeider [c]

[a] *Linköping University, Linköping, Sweden*
[b] *Masaryk University, Botanická 68a, 60200 Brno, Czech Republic*
[c] *Vienna University of Technology, Vienna, Austria*

A R T I C L E   I N F O

Article history:
Received 24 October 2013
Received in revised form 25 October 2014
Accepted 18 March 2015
Available online 21 April 2015

Keywords:
Complexity of automated planning
Parameterized complexity
Kernelization

A B S T R A C T

The propositional planning problem is a notoriously difficult computational problem, which remains hard even under strong syntactical and structural restrictions. Given its difficulty it becomes natural to study planning in the context of parameterized complexity. In this paper we continue the work initiated by Downey, Fellows and Stege on the parameterized complexity of planning with respect to the parameter "length of the solution plan." We provide a complete classification of the parameterized complexity of the planning problem under two of the most prominent syntactical restrictions, i.e., the so called PUBS restrictions introduced by Bäckström and Nebel and restrictions on the number of preconditions and effects as introduced by Bylander. We also determine which of the considered fixed-parameter tractable problems admit a polynomial kernel and which do not.

1. Introduction

The (propositional) planning problem has been the subject of intensive study in knowledge representation, artificial intelligence and control theory and is relevant for a large number of industrial applications including automated control of industrial processes, natural language processing, and system verification [33]. In the planning problem one is given a set of *variables* (over a given domain), a set of *actions* (or *operators*), an *initial state*, and a *goal state*. Hereby, the initial state is a *total state*, i.e., an assignment of all variables to some domain value, and the goal state is a *partial state*, i.e., an assignment of some subset of the variables to some domain value. Furthermore, every action consists of two partial states: the partial state representing the *preconditions*, which specifies the conditions that have to hold before the action can be executed, and the partial state representing the *post-conditions* (or *effects*), which provides the conditions that will hold after the action has been executed. Then, the planning problem is to decide whether there is a *plan*, i.e., a sequence of actions that can be

☆ Parts of this paper appeared in a preliminary and shortened form in the Proceedings of AAAI 2012, the 26th AAAI Conference on Artificial Intelligence [4] and in the Proceedings of CIAC 2013, the 8th International Conference on Algorithms and Complexity [9]. Ordyniak and Szeider's research was supported by the European Research Council, grant reference 239962. Ordyniak's research was partially supported by the Employment of Newly Graduated Doctors of Science for Scientific Excellence (CZ.1.07/2.3.00/30.0009). Szeider's research was partially supported by the Austrian Science Fund, Project P 26200.

* Corresponding author.
E-mail addresses: christer.backstrom@liu.se (C. Bäckström), peter.jonsson@liu.se (P. Jonsson), sordyniak@gmail.com (S. Ordyniak), stefan@szeider.net (S. Szeider).

http://dx.doi.org/10.1016/j.jcss.2015.04.002

applied to the initial state and result in the goal state. It is known that the problem of deciding whether a plan exists or not is **PSPACE**-complete [12,18].

What we have just described is usually referred to as the SAS$^+$ formalism [12], and it is formally defined in Section 3. This can be viewed as a generalization of the classical STRIPS formalism [18], which uses binary variable domains only. It is known that planning is computationally equivalent in the two formalisms in the general case [3].

Although various **NP**-complete and even polynomial-time tractable restrictions are known in the literature [12,17,18,42] these are often considered not to coincide well with cases that are interesting in practice. Classical complexity analysis is often too coarse to give relevant results for planning, since most interesting restrictions seem to remain **PSPACE**-complete. Despite this, there has been very few attempts to use alternative analysis methods. The few exceptions include probabilistic analysis [19], approximation [13,41] and padding [7].

Another obvious alternative is to use the framework of Parameterized Complexity which offers the more relaxed notion of *fixed-parameter tractability* (FPT). A problem is fixed-parameter tractable if it can be solved in time $f(k)n^{O(1)}$ where f is an arbitrary function of the parameter k (which measures some aspect of the input) and n is the input size. **FPT** denotes also the class of all fixed-parameter tractable decision problems. The Weft hierarchy is formed by the sequence of parameterized complexity classes $\mathbf{FPT} \subseteq \mathbf{W}[1] \subseteq \mathbf{W}[2] \subseteq \cdots$ where all inclusions are believed to be strict. Thus, by showing that a problem is $\mathbf{W}[i]$-complete for some $i \geq 1$ one obtains strong theoretical evidence that the problem is not fixed-parameter tractable.

Already in a 1999 paper, Downey, Fellows and Stege [24] initiated the parameterized analysis of planning, taking the minimum number of steps from the initial state to the goal state (i.e., the length of the solution plan) as the parameter. However, the parameterized viewpoint did not immediately gain momentum for analysis of planning and it is only during the last few years that we have witnessed a strongly increased interest in this method, cf. [7,10,23,45].

In this article, we use the same parameter as Downey et al. and provide a complete analysis of planning under various syntactical restrictions, in particular under:

1. *Bylander's restrictions*, which are given in terms of upper bounds on the number of preconditions and effects of actions, as considered by Bylander [18], i.e., the number of variables whose value is specified in a precondition or in an effect of an action, respectively, and the
2. *PUBS restrictions*, which are combinations of the four properties of being *post-unique* (P), *unary* (U), *binary* (B), and *single-valued* (S), as considered by Bäckström and Nebel [12]. Informally, post-unique means that for every variable/value-pair there is at most one action whose effect sets the variable to the given value, unary means that every action has an effect on at most one variable, binary means that the domain of every variable has at most two values, and single-valued means that if two actions have a precondition on some common variable that they do not change, then they require the same value from that variable (see Section 3 for exact definitions).

These restrictions were among the first attempts to understand why and when planning is hard or easy and they have had a heavy influence on later theoretical research in planning.

It is known that a decidable parameterized problem P is fixed-parameter tractable if and only if it admits a polynomial-time reduction to a decidable parameterized problem Q where the size of the resulting instance is bounded by a function f of the parameter [30,32,35]. The function f is called the *bi-kernel size* (or *kernel-size* if $P = Q$). By providing upper and lower bounds on the (bi-)kernel size, one can rigorously establish the potential of polynomial-time preprocessing for the problem at hand. We examine whether the fixed-parameter tractable subcases of planning problems, which we obtain, admit polynomial (bi-)kernels or not.

Our results

We obtain a full classification of the parameterized complexity of planning with respect to the length of the solution plan, under Bylander's restrictions and the PUBS restrictions.

Table 1 displays the complexity results under *Bylander's restrictions* (for arbitrary domain sizes ≥ 2) under both parameterized and classical analysis. The parameterized results in Table 1 are derived as follows. For actions with an arbitrary number of effects, the results follow from Theorems 1 and 4. For actions with at most one effect, we have two cases: With no preconditions the problem is trivially in **P**. Otherwise, the results follow from Theorems 2 and 5. The case where the number of effects is bounded by some constant $e > 1$ and the number of preconditions p is at least 1 can be reduced in polynomial time to the case with only one effect using a reduction by Bäckström [2, proof of Theorem 6.7]. Since this reduction is a parameterized reduction we have membership in **W**[1] by Theorem 5. In this case we also obtain **W**[1]-hardness by Theorem 2. For the final case ($p = 0$), we obtain **W**[1]-hardness from Theorem 3 and containment in **W**[1] from Theorem 5 if the number of effects e is at least 3. The case where also $e = 2$ is fixed-parameter tractable according to Theorem 7.

The complexity results for the *PUBS restrictions* are displayed in Fig. 1. Solid lines denote separation results by Bäckström and Nebel [12], using standard complexity analysis, while dashed lines denote separation results from our parameterized analysis. The **W**[2]-completeness results follow from Theorems 1 and 4, the **W**[1]-completeness results follow from Theorems 2 and 5, and the **FPT** results follow from Corollary 2. Since **W**[1] and **W**[2] are not directly comparable to the standard complexity classes we get interesting separations from combining the two methods. For instance, we can single out restriction U as making planning easier than in the general case, which is not possible under standard analysis. Since planning