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Fault-tolerant vertex-pancyclicity of locally twisted cubes LTQ_n [☆]

Xirong Xu [a,*], Yazhen Huang [a], Peng Zhang [b], Sijia Zhang [a]

[a] School of Computer Science and Technology, Dalian University of Technology, Dalian, 116024, PR China

[b] School of Computer and Information Engineering, Zhongshan College of Dalian Medical University, Dalian, 116085, PR China

HIGHLIGHTS

- The structure of LTQ_n and some definitions and notations.
- We introduce some properties of LTQ_n.
- Investigates the fault-tolerant vertex-pancyclicity of LTQ_n.
- Prove that LTQ_n is n-3 fault-tolerant 4-vertex-pancyclicity.

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ABSTRACT

The n-dimensional locally twisted cube LTQ_n is a variant of the hypercube, which possesses some properties superior to the hypercube. This paper investigates the fault-tolerant vertex-pancyclicity of LTQ_n, and shows that if LTQ_n ($n \geqslant 3$) contains at most $n-3$ faulty vertices and/or edges then, for any fault-free vertex u and any integer ℓ with $4 \leqslant \ell \leqslant 2^n - f_v$ except for 5, there is a fault-free cycle of length ℓ containing the vertex u, where f_v is the number of faulty vertices. The result is optimal in some senses.

1. Introduction

Interconnection networks play an important role in parallel processing systems. An interconnection network can be represented by a graph $G = (V, E)$, where V represents the vertex set and E represents the edge set. The capacity of embedding other existing network into an interconnection network is a critical issue in evaluating an interconnection network. Suppose that some process can be naturally decomposed into a collection of subprocesses that can be executed concurrently with certain communication among subprocesses. One obtains a graph by denoting each subprocess by a vertex and each communication between subprocesses by an edge between the corresponding vertices. The problem of allocating the subprocesses to processors in the given network can be modeled by the following graph embedding problem: given a host graph $G_2 = (V_2, E_2)$, which represents the network into which other networks are to be embedded, and a guest graph $G_1 = (V_1, E_1)$, which represents the network to be embedded, the problem is to find a mapping from each node of G_1 to a node of G_2, and a mapping from each edge of G_1 to a path in G_2 [27]. One common measure of effectiveness of an embedding is the dilation. The dilation of embedding ψ is defined as $\mathrm{dil}(G_1, G_2, \psi) = \max\{\mathrm{dist}(G_2, \psi(u), \psi(v)) | (u, v) \in E_1\}$, where $\mathrm{dist}(G_2, \psi(u), \psi(v))$ denotes the distance between the two nodes $\psi(u)$ and $\psi(v)$ in G_2. The smaller the dilation of an embedding is, the shorter the communication delay that the graph G_2 simulates the graph G_1 [1]. As two common guest graphs, linear arrays (i.e. paths) [9,8,7] and rings (i.e. cycles) [2,6,15] are two fundamental networks for parallel and distributed computing.

In large interconnection networks, nodes or edges tend to become faulty. It is important to find an embedding of a guest graph into a host graph where all faulty nodes and edges have been removed. This is called fault-tolerant embedding. Much work has been done on the fault-tolerant embedding [3,10,12,19,21,20,17,4,13].

The locally twisted cube has many properties superior to hypercube. Though both the locally twisted cube and the ordinary hypercube have the same number of vertices and the same vertex degree, the diameter of the locally twisted cube is approximately half that of the ordinary hypercube.

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* Corresponding author.
E-mail address: xirongxu@dlut.edu.cn (X. Xu).

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In this paper, we are interested in the path and/or cycle embedding properties of the n-dimensional locally twisted cube LTQ_n. Yang et al. proposed this new network [26] and proved that LTQ_n contains cycles of all lengths from 4 to 2^n [27]. Ma, Xu [23] and Hu et al. [16], independently, improved this result by proving that for any edge in LTQ_n there are cycles of all lengths containing it. Ma and Xu [22] further improved this result by showing that for any two different vertices x and y with distance d in LTQ_n, there exist xy-paths of all lengths from d to $2^n - 1$ except for $d + 1$.

Even when faulty elements occur, Chang et al. [5] and Park et al. [24], independently, showed that LTQ_n still contains fault-free cycles of all lengths provided that faulty elements do not exceed $n - 2$.

Very recently, Han et al. [11] have showed that LTQ_n with at most $n-3$ faulty elements contains paths of all lengths from $2^{n-1} - 1$ to $2^n - f_v - 1$ between any two distinct fault-free vertices, where f_v is the number of faulty vertices. Hsieh and Wu [14] have considered more faulty edges and showed that LTQ_n contains a fault-free Hamiltonian cycle provided that faulty edges do not exceed $2n - 5$ and each vertex is incident with at least two fault-free edges. This condition is natural since, in practical applications, the probability is small for a vertex x being isolated (all links incident with x are faulty) or pendant (only one link incident with x is fault-free and the others are all faulty).

In this paper, we show that for any vertex $u \in V(LTQ_n)$ and any integer $4 \leqslant \ell \leqslant 2^n - f_v$ except for 5, there exists cycle C of length ℓ in LTQ_n such that u is in C if $f_e + f_v \leqslant n - 3$. The approach we use is based on the recursive construction of LTQ_n.

The remaining part of this paper is organized as follows. In Section 2, we recall the structure of LTQ_n, and some definitions and notations. In Section 3, we introduce some properties of LTQ_n to be used in our proofs. In Section 4, we give the proof of our result. Finally, we give some concluding remarks in Section 5.

2. Preliminaries

In this section, we will give some definitions and properties about LTQ_n. A graph $G = (V, E)$ consists of a vertex-set V and an edge-set E, where $V = V(G)$ is a finite set and $E = E(G)$ is a subset of xy–xy is an unordered pair of V. Two vertices x and y are adjacent if xy is an edge of G, and which are also the end-vertices of xy. For a vertex x, the vertex adjacent to x is called as the neighbor of x. The degree of a vertex x is the number of edges incident with it. A graph is called k-regular if each vertex has degree k. For two distinct vertices x and y, an xy-path between x and y is a sequence of distinct vertices in which any two consecutive vertices are adjacent. The length of a path is the number of edges on the path. An xy-path of length at least three is called a cycle if $x = y$. A connected subgraph of G is called a spanning tree if it contains all vertices of G and no cycles, in which a distinguished vertex is called the root of the spanning tree.

The distance between two distinct vertices x and y in G is the length of a shortest xy-path in G, and the diameter of G is the maximum distance between any two vertices.

A non-empty subset M of $E(G)$ is called a matching of G if no two of its elements have a common end vertices in G. A matching M is perfect if every vertex of G is an end-vertex of some edge in M.

We now recall the definition of the n-dimensional locally twisted cube, proposed by Yang, Evans and Megson [26], which has 2^n vertices, and each vertex is an n-string on $\{0, 1\}$.

Definition 1 (*[26]*)**.** The n-dimensional locally twisted cube, denoted by LTQ_n ($n \geqslant 2$), is recursively defined as follows.

(1) LTQ_2 is a graph isomorphic to Q_2.
(2) For $n \geqslant 3$, LTQ_n is built from disjoint copies of LTQ_{n-1} according to the following steps. Let LTQ^0_{n-1} and LTQ^1_{n-1} denote graphs obtained by prefixing labels of each vertex of one copy of LTQ_{n-1} with 0 and with 1, respectively, and connect a vertex $x = 0x_2x_3 \ldots x_n$ of LTQ^0_{n-1} with another vertex $y = 1(x_2 + x_n)x_3 \ldots x_n$ of LTQ^1_{n-1} by an edge xy, where '+' represents the modulo 2 addition.

The graphs shown in Fig. 1 are LTQ_3 and LTQ_4. The locally twisted cube LTQ_n can be equivalently defined with the following non-recursive fashion.

Definition 2 (*[26]*)**.** For $n \geqslant 2$, the n-dimensional locally twisted cube LTQ_n is a graph with n-strings on $\{0, 1\}$ as the vertex set. Two vertex $x = x_1x_2 \ldots x_{n-1}x_n$ and $y = y_1y_2 \ldots y_{n-1}y_n$ of LTQ_n are adjacent if and only if either

(a) $x_i = \bar{y}_i$ and $x_{i+1} = y_{i+1} + x_n$ for some $1 \leqslant i \leqslant n-2$, and $x_j = y_j$ for all the remaining bits, where '+' represents the modulo 2 addition, or
(b) $x_i = \bar{y}_i$ for some $i \in \{n-1, n\}$, and $x_i = y_i$ for all the remaining bits.

According to the above definition, it is not difficult to see that LTQ_n is an n-regular graph with 2^n vertices and $n2^{n-1}$ edges. From the definition, LTQ_n can be expressed as the union of two disjoint copies of LTQ_{n-1} by adding a perfect matching between them according to the specified rule. For short, we often write $LTQ_n = L \oplus R$, where $L \cong LTQ^0_{n-1}$ and $R \cong LTQ^1_{n-1}$.

We now make some remarks on the n-dimensional locally twisted cube.

Firstly, like to many variants of the hypercube such as the twisted cube, the crossed cube, the augmented cube and otherwise, the locally twisted cube not only keeps many nice properties of the hypercube such as regularity, high connectivity and high recursive constructability, but also has diameter of about half of that of the hypercube of the same size.

Secondly, the locally twisted cube also keeps a nice property of the hypercube, that is, the labels of any two adjacent vertices differ in at most two successive bits. However, a common feature of the above-mentioned variants is that the labels of some neighbor vertices may differ in a large number of bits. As a result, a portion of good properties of hypercube are lost in these variants. For example, the design of efficient parallel algorithms on these variants turns out to be a difficult task [26].

Thirdly, the locally twisted cube LTQ_n contains cycles of all lengths from 4 to 2^n [27], but the hypercube Q_n contains only even cycles since it is a bipartite graph. Thus, LTQ_n is superior to Q_n in cycle embedding property.

Fourthly, the construction of the locally twisted cube LTQ_n is quite different from that of the twisted cube TQ_n. The former is defined for any positive integer n, while the latter only for odd integer.

Lastly, it should be noted that, like to many variants of the hypercube, the locally twisted cube LTQ_n is not vertex-transitive for $n \geqslant 4$ proved by Liu et al. [18].

3. Properties

In this section, we introduce some properties of LTQ_n to be used in our proofs in Section 4.

Yang, Evans and Megson [26] found an isomorphic expression of LTQ_n. For example, two graphs shown in Fig. 2 are other expressions of LTQ_3 and LTQ_4, respectively.

In general, they proved the following result.