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A note on the complexity of minimum latency data aggregation scheduling with uniform power in physical interference model

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ABSTRACT

In this paper we prove that the Minimum Latency Aggregation Scheduling (MLAS) problem in the Signal-to-Interference-Noise-Ratio (SINR) model is APX-hard in the uniform power model.

1. Introduction

Data aggregation is one of main applications of Wireless Sensor Networks (WSNs), and its main purpose is to collect data periodically from the sensor nodes and forward it to a destination called the sink node. As these tiny devices have limited energy resources, researchers have focused on finding ways to avoid sensor nodes' unnecessary retransmissions of their collected data in order to extend the network lifetime. One approach is to compute schedules with the minimum number of *timeslots* such that data can be aggregated without any collision or interference. This problem is known as the Minimum Latency Aggregation Scheduling (MLAS) problem.

In the literature, wireless networks are commonly modeled as graphs where any two nodes are connected via a communication edge if they are covered by each other's transmission range. When considering the MLAS problem on such networks, choosing the *interference model* is a crucial step. While a substantial amount of research results have been obtained for the *graph-based interference model*, recently, several researchers have started investigating the problems in the more realistic *physical interference model*, also known as Signal-to-Interference-Noise-Ratio (SINR), which, unlike the graph model, more adequately captures real world phenomena. As the SINR model has been introduced only recently, few works exist and algorithms that guarantee theoretical performances are scarce.

Along with the interference models, researchers have adopted one of two power models, *uniform power* and *non-uniform power* models, concerning the MLAS problem. The uniform power model assumes *no power control*, i.e., a uniform power level is typically used, whereas in the non-uniform model, determining the right power levels to be assigned to sending nodes

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could also be part of the problem (also known as *power control*). The non-uniform power model is divided into three cases: the *bounded power*, the *unlimited power*, or the *discrete power* model. In the bounded power model, each node u is assigned a transmission power level $p_u \in [p_{min}, p_{max}]$, and in the unlimited power model, u is assigned a transmission power level $p_u \in [p_{min}, \infty]$. In the discrete power model, each node u is assigned a transmission power level $p_u \in \{p_1, p_2, ..., p_k\}$, where k is the number of power levels used in the network.

The MLAS problem in the graph-based interference model has been investigated by many researchers over the last several years. Assuming the uniform power model, in the *collision-free graph model*, Chen et al. [3] proved the NP-hardness of the MLAS problem and proposed a $(\Delta - 1)$-approximation algorithm, where Δ is the maximum node degree. Later, Huang et al. [6] proposed a nearly-constant factor approximation algorithm whose latency is bounded by $23R + \Delta - 18$, and Yu et al. [15] introduced a distributed algorithm whose latency is bounded by $24D + 6\Delta + 16$, where R is the radius and D is the diameter of the network. Subsequently, Xu et al. [13,14] introduced a better constant factor approximation algorithm whose latency is bounded by $16R + \Delta - 14$, and Wan et al. [12] proposed three algorithms whose latency is bounded by $15R + \Delta - 4$, $2R + O(\log R) + \Delta$, and $(1 + O(\frac{\log R}{\sqrt[3]{R}}))R + \Delta$, respectively. While only collision was considered in these papers, some researchers have studied the problem taking into consideration interference as well. This is done in the *collision-interference-free graph model*. Wan et al. [12], An et al. [1] proposed constant factor approximation algorithms whose latency is bounded by $O(R + \Delta)$. In the SINR model, Li et al. [10] introduced the first constant factor approximation algorithm whose latency is bounded by $O(R + \Delta)$.

Assuming the nonuniform power model, An et al. [1] proved an $\Omega(\log n)$ approximation lower bound in the metric model, where n is the number of nodes. It was investigated without power control in the collision-interference-free graph model with discrete power levels. In the SINR model with bounded power, Lam et al. [8] studied the MLAS problem with power control, and showed the first constant factor approximation algorithm whose latency is bounded by $O(R + \log n)$. Later, Du et al. [4] proposed another constant factor approximation algorithm whose latency is also bounded by $O(R + \log n)$ in the same model. In the unbounded power model with power control, Li et al. [9] proposed a distributed algorithm that yields $O(\chi)$ timeslots, where χ is the link length diversity, and a centralized algorithm whose latency is $O(\log^3 n)$ which was improved by Halldórsson and Mitra [5] to $O(\log n)$. In the discrete power model without power control, Lam et al. [7] showed not only an $\Omega(\log n)$ approximation lower bound in the metric SINR model, but also its NP-hardness in the geometric SINR model. Lam et al. [7] has been extended in An et al. [2] and introduced two constant factor approximation algorithms whose latencies are bounded by $O(R + \Delta)$ assuming the dual power model, i.e., each node is assigned either the high power level or the low power level.

In this paper, we continue the study of the Minimum Latency Aggregation Scheduling (MLAS) problem in the metric SINR model with discrete power levels, but without power control. Assuming the most restricted model of uniform power, we prove the APX-hardness of the problem in the metric SINR model.

The rest of this paper is organized as follows. In Section 2, we describe our network model and introduce the definitions used in this paper. In Section 3, we prove the APX-hardness. Finally, Section 4 contains some concluding remarks.

2. Models and definitions

In this section, we introduce two SINR models, the metric model and the geometric model. For the former, we model a WSN in a metric space as (V, D), where V is a set of sensor nodes, and $D : V \times V \longrightarrow R^+$ is the distance function that satisfies the triangle inequality.

Considering a communication link $l_i(s_i, r_i)$, where s_i is a sender and r_i is a receiver, let $CL(l_i)$ be the set of other links concurrently sending at the time as l_i. According to the physical interference model, we have

$$SINR(l_i) = \frac{p_{s_i} D(s_i, r_i)^{-\alpha}}{N + \sum_{l_j \in CL(l_i)} p_{s_j} D(s_j, r_i)^{-\alpha}}$$

where N is the ambient noise, α is the path loss, and p_u is the power level assigned to node u. Then, the receiver r_i can successfully receive the signal from the sender s_i if and only if its SINR value exceeds a given threshold $\beta \geq 1$. So a node u with power p_u can send signals to only nodes in the distance d where $d^\alpha \leq \frac{p_u}{N\beta}$. We call these nodes u's neighbors.

In this paper, we are specifically concerned with the uniform power model. We use a single transmission power level denoted by p where each node u in V uses the transmission power p to communicate.

In the restricted geometric model, the set V of sensor nodes are deployed on the plane and the distance function D is defined as the Euclidean distance between two nodes. Regarding the definition of neighboring nodes, we assume that a sender u can send data to only nodes in the distance d, where $d^\alpha \leq \frac{p_u}{\gamma\beta N}$, for some constant $\gamma > 1$.

The data aggregation problem for either model is defined as follows. A schedule is defined to be a sequence of timeslots, at each of which, several nodes are scheduled to send its aggregated data to one of its neighbors, and every node can be scheduled as a sender only once. Formally, at each timeslot t, we have an assignment vector $\pi_t = (l_{t_1}, l_{t_2}, ..., l_{t_w})$, in which l_{t_i} is a directed link from s_{t_i} to r_{t_i} satisfying the SINR threshold inequality. And a schedule, as a sequence of assignment vectors, is denoted as $\Pi = (\pi_1, \pi_2, ..., \pi_L)$, where L is the length of schedule or the schedule latency.

Given a set of source nodes and the sink node s, the objective of the data aggregation problem is to find the minimum latency schedule to aggregate data from all source nodes to the given sink.