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Kernelization complexity of possible winner and coalitional manipulation problems in voting

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ABSTRACT

In the Possible winner problem in computational social choice theory, we are given a set of partial preferences and the question is whether a distinguished candidate could be made winner by extending the partial preferences to linear preferences. Previous work has provided, for many common voting rules, fixed parameter tractable algorithms for the Possible winner problem, with number of candidates as the parameter. However, the corresponding kernelization question is still open and in fact, has been mentioned as a key research challenge [10]. In this paper, we settle this open question for many common voting rules.

We show that the Possible winner problem for maximin, Copeland, Bucklin, ranked pairs, and a class of scoring rules that includes the Borda voting rule does not admit a polynomial kernel with the number of candidates as the parameter. We show however that the Coalitional manipulation problem which is an important special case of the Possible winner problem does admit a polynomial kernel for maximin, Copeland, ranked pairs, and a class of scoring rules that includes the Borda voting rule, when the number of manipulators is polynomial in the number of candidates. A significant conclusion of our work is that the Possible winner problem is harder than the Coalitional manipulation problem since the Coalitional manipulation problem admits a polynomial kernel whereas the Possible winner problem does not admit a polynomial kernel.

1. Introduction

In many real life situations including multiagent systems, agents often need to aggregate their preferences and agree upon a common decision (candidate). Voting is an immediate natural tool in these situations. Common and classical applications of voting rules in artificial intelligence include collaborative filtering [33], planning among multiple automated agents [20], etc.

Usually, in a voting setting, it is assumed that the votes are complete orders over the candidates. However, due to many reasons, for example, lack of knowledge of voters about some candidates, a voter maybe indifferent between some pairs of candidates. Hence, it is both natural and important to consider scenarios where votes are partial orders over the candidates. When votes are only partial orders over the candidates, the winner cannot be determined with certainty since it depends on how these partial orders are extended to linear orders. This leads to a natural computational problem

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called the Possible winner [27] problem: given a set of partial votes P and a distinguished candidate c, is there a way to extend the partial votes to linear ones to make c win? The Possible winner problem has been studied extensively in the literature [29,34,35,37,8,6,14,5,2,28,23] following its definition in [27]. The Possible winner problem is known to be NP-complete for many common voting rules, for example, scoring rules, maximin, Copeland, Bucklin, and ranked pairs etc. [37]. Walsh [35] showed, for a constant number of candidates, that the Possible winner problem can be solved in polynomial time for all the voting rules mentioned above. An important special case of the Possible winner problem is the Coalitional manipulation problem [1] where only two kinds of partial votes are allowed – complete preference and empty preference. The set of empty votes is called the manipulators' vote and is denoted by M. The Coalitional manipulation problem is NP-complete for maximin, Copeland, and ranked pairs voting rules even when $|M| \geq 2$ [21,22,38]. The Coalitional manipulation problem is in P for the Bucklin voting rule [38]. We refer to [37,35,38] for detailed overviews.

1.1. Our methodology

Preprocessing, as a strategy for coping with hard problems, is universally applied in practice. The main goal here is *instance compression* – the objective is to output a smaller instance while maintaining equivalence. In the classical setting, NP-hard problems are unlikely to have efficient compression algorithms (since repeated application would lead to a polynomial time algorithm for the problem, which would imply P = NP). However, the breakthrough notion of *kernelization* in parameterized complexity provides a mathematical framework for analyzing the quality of preprocessing strategies. In parameterized complexity, each problem instance (x, k) comes with a parameter k. The parameterized problem is said to admit a *kernel* if there is a polynomial time algorithm (where the degree of polynomial is independent of k), called a *kernelization algorithm*, that reduces the input instance to an instance with size bounded by a function of k, while preserving the answer. This has turned out to be an important and widely applied notion in theory, and has also proven very successful in practice [36,30]. Quantitatively, running a kernelization algorithm before solving it using an algorithm that runs in time $f(|x|)$ brings down the running time to $f(k) + p(|x|)$, where $|x|$ is the size of the input instance and the running time of the kernelization algorithm is $p(|x|)$.

A problem with parameter k is called *fixed parameter tractable* (FPT) if it is solvable in time $f(k) \cdot p(|x|)$, where f is an arbitrary function of k and p is a polynomial in the input size $|x|$. The existence of a fixed parameter tractable algorithm implies existence of a kernel for that problem. However, the size of the kernel need not be polynomial in the parameter. A polynomial kernel is said to exist if there is a kernelization algorithm that can output an equivalent problem instance of size polynomial in the parameter. We refer to [19,32] for an excellent overview on fixed parameter algorithms and kernelization.

1.2. Contributions

Discovering kernelization algorithms is currently an active and interesting area of research in computational social choice theory [5,3,7,12,25,11,4,18]. Betzler et al. [8] showed that the Possible winner problem admits fixed parameter tractable algorithm when parameterized by the total number of candidates for scoring rules, maximin, Copeland, Bucklin, and ranked pairs voting rules. Yang et al. [40,39] provides efficient fixed parameter tractable algorithms for the Coalitional manipulation problem for the Borda, maximin, and Copeland voting rules. A natural and practical follow-up question is whether the Possible winner and Coalitional manipulation problems admit a polynomial kernel when parameterized by the number of candidates. This question has been open ever since the work of Betzler et al. and in fact, has been mentioned as a key research challenge in parameterized algorithms for computational social choice theory [10]. Betzler et al. showed non-existence of polynomial kernel for the Possible winner problem for the k-approval voting rule when parameterized by (t, k), where t is the number of partial votes [3]. The NP-complete reductions for the Possible winner problem for scoring rules, maximin, Copeland, Bucklin, and ranked pairs voting rules given by Xia et al. [38] are from the Exact 3 set cover problem. Their results do not throw any light on the existence of a polynomial kernel since Exact 3 set cover has a trivial $O(m^3)$ kernel where m is the size of the universe. In our work in this paper, we show that there is no polynomial kernel (unless CoNP ⊆ NP/Poly) for the Possible winner problem, when parameterized by the total number of candidates, with respect to maximin, Copeland, Bucklin, and ranked pairs voting rules, and a class of scoring rules that includes the Borda voting rule. These hardness results are shown by a parameter-preserving many-to-one reduction from the Small universe set cover problem for which there does not exist any polynomial kernel parameterized by universe size unless CoNP ⊆ NP/Poly [17].

On the other hand, we show that the Coalitional manipulation problem admits a polynomial kernel for maximin, Copeland, and ranked pairs voting rules, and a class of scoring rules that includes the Borda voting rule when we have *poly*(m) number of manipulators – specifically, we exhibit an $O(m^2|M|)$ kernel for maximin and Copeland voting rules, and an $O(m^4|M|)$ kernel for the ranked pairs voting rule, where m is the number of candidates and M is the set of manipulators. The Coalitional manipulation problem for the Bucklin voting rule is in P [38] and thus the kernelization question does not arise.

A significant conclusion of our work is that, although the Possible winner and Coalitional manipulation problems are both NP-complete for many voting rules, the Possible winner problem is harder than the Coalitional manipulation