Theoretical Computer Science 602 (2015) 125–131

Contents lists available at ScienceDirect

Theoretical Computer Science

www.elsevier.com/locate/tcs

Pancyclicity and bipancyclicity of folded hypercubes with both vertex and edge faults

Che-Nan Kuo

Department of Animation and Game Design, TOKO University, No. 51, Sec. 2, University Road, Pu-Tzu City, ChiaYi County 61363, Taiwan

ARTICLE INFO

Article history:
Received 18 May 2015
Received in revised form 24 July 2015
Accepted 20 August 2015
Available online 28 August 2015
Communicated by S.-y. Hsieh

Keywords:
Interconnection networks
Folded hypercubes
Pancyclicity
Bipancyclicity
Fault-tolerant
Fault-free

ABSTRACT

A graph is said to be pancyclic if it contains cycles of every length from its girth to its order inclusive; and a bipartite graph is said to be bipancyclic if it contains cycles of every even length from its girth to its order. The pancyclicity or the bipancyclicity of a given network is an important factor in determining whether the network's topology can simulate cycles of various lengths. An n-dimensional folded hypercube FQ_n is a well-known variation of an n-dimensional hypercube Q_n which can be constructed from Q_n by adding an edge to every pair of vertices with complementary addresses. FQ_n for any odd n is known to bipartite. In this paper, let FF_v and FF_e denote the sets of faulty vertices and faulty edges in FQ_n. Then, we consider the pancyclicity and bipancyclicity properties in $FQ_n - FF_v - FF_e$, as follows:

1. For $n \geq 3$, $FQ_n - FF_v - FF_e$ contains a fault-free cycle of every even length from 4 to $2^n - 2 \cdot |FF_v|$, where $|FF_v| + |FF_e| \leq n - 1$;
2. For $n \geq 4$ is even, $FQ_n - FF_v - FF_e$ contains a fault-free cycle of every odd length from $n+1$ to $2^n - 2 \cdot |FF_v| - 1$, where $|FF_v| + |FF_e| \leq n - 1$.

1. Introduction

Choosing an appropriate *interconnection network* (*network* for short) is an important integral part of designing parallel processing and distributed systems. Many network topologies have been proposed [2,15,26]. Among the proposed network topologies, the *hypercube* [3] is a well-known network model. The hypercube has several excellent properties such as recursive structure, regularity, symmetry, small diameter, short mean internode distance, low degree, and much smaller edge complexity, which are very important for designing massively parallel or distributed systems [18]. Numerous variants of the hypercube have been proposed in the literature [5,6,23]. One variant that has been the focus of a great deal of research is the *folded hypercube*, which can be constructed from a hypercube by adding an edge to every pair of vertices that are the farthest apart, i.e., two vertices with complementary addresses. The folded hypercube has been shown to be able to improve the system's performance over a regular hypercube in many measurements, such as diameter, fault diameter, connectivity, and so on [5,24].

An important feature of an interconnection network is its ability to efficiently simulate algorithms designed for other architectures. Such a simulation can be formulated as *network embedding*. An *embedding* of a *guest network* G into a *host*

E-mail address: cn.kuo@mail.toko.edu.tw.

http://dx.doi.org/10.1016/j.tcs.2015.08.018

network H is defined as a one-to-one mapping f from the vertex set of G to the vertex set of H. Under f, an edge in G corresponds to a path in H [18]. Cycles (rings), the most fundamental networks for parallel and distributed computation, are suitable for designing simple algorithms with low communication costs. Numerous efficient algorithms designed on rings for solving various algebraic problems and graph problems can be found in [1,18].

Since faults may occur when a network is put into use, it is practically meaningful and important to consider faulty networks. Previously, the problem of fault-tolerant cycle embedding on an n-dimensional folded hypercube FQ_n has been studied in [4,7,9,10,13,14,16,17,20,24,28]. Let FF_v and FF_e denote the sets of faulty vertices and faulty edges in FQ_n. Hsieh [10] showed that $FQ_n - FF_v - FF_e$ contains a fault-free cycle of length at least $2^n - 2 \cdot |FF_v|$, where $n \geq 3$. In this paper, we extend Hsieh's [10] result to obtain two further properties, which consider both vertex and edge faults, as follows:

1. For $n \geq 3$, $FQ_n - FF_v - FF_e$ contains a fault-free cycle of every even length from 4 to $2^n - 2 \cdot |FF_v|$, where $|FF_v| + |FF_e| \leq n - 1$;
2. For $n \geq 4$ is even, $FQ_n - FF_v - FF_e$ contains a fault-free cycle of every odd length from $n + 1$ to $2^n - 2 \cdot |FF_v| - 1$, where $|FF_v| + |FF_e| \leq n - 1$.

Throughout this paper, a number of terms—network and graph, node and vertex, edge and link—are used interchangeably. The remainder of this paper is organized as follows: in Section 2, we provide some necessary definitions and notations. Sections 3 and 4 present our main results of embedding cycles with even lengths and cycles with odd lengths, respectively. Some concluding remarks are given in Section 5.

2. Preliminaries

A graph $G = (V, E)$ is an ordered pair in which V is a finite set and E is a subset of $\{(u, v)|(u, v)$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set*. We also use $V(G)$ and $E(G)$ to denote the vertex set and the edge set of G, respectively. Two vertices u and v are *adjacent* if $(u, v) \in E$. A graph $G = (V_0 \cup V_1, E)$ is bipartite if $V_0 \cap V_1 = \emptyset$ and $E \subseteq \{(x, y)|x \in V_0$ and $y \in V_1\}$. A path $P[v_0, v_k] = \langle v_0, v_1, \ldots, v_k\rangle$ is a sequence of distinct vertices in which any two consecutive vertices are adjacent. We call v_0 and v_k the *end-vertices* of the path. In addition, a path may contain a *subpath*, denoted as $\langle v_0, v_1, \ldots, v_i, P[v_i, v_j], v_j, v_{j+1}, \ldots, v_k\rangle$, where $P[v_i, v_j] = \langle v_i, v_{i+1}, \ldots, v_{j-1}, v_j\rangle$. The length of a path is the number of edges on the path. A path $\langle v_0, v_1, \ldots, v_k\rangle$ forms a *cycle* if $v_0 = v_k$ and $v_0, v_1, \ldots, v_{k-1}$ are distinct. A vertex is *fault-free* if it is not faulty. An edge is *fault-free* if the two end-vertices and the edge between them are not faulty. Vertex u is a *fault-free* adjacent vertex of v if u and (u, v) are not faulty. A path (cycle) is *fault-free* if it contains no faulty edges and faulty vertices.

Usually when the Hamiltonicity of a graph G is concerned, it is investigated whether G is Hamiltonian or Hamiltonian-connected. A cycle (respectively, path) in G is called a *Hamiltonian cycle* (respectively, *Hamiltonian path*) if it contains every vertex of G exactly once. A graph G is *Hamiltonian* if it contains a Hamiltonian cycle, and *Hamiltonian-connected* if there exists a Hamiltonian path between every two distinct vertices of G. A bipartite graph G is *Hamiltonian-laceable* if there exists a Hamiltonian path between any two vertices from different partite sets. A Hamiltonian-laceable graph $G = (V_0 \cup V_1, E)$ is *strong* [8] if there is a simple path of length $|V_0| + |V_1| - 2$ between any two vertices of the same partite set. A Hamiltonian-laceable graph $G = (V_0 \cup V_1, E)$ is *hyper-Hamiltonian laceable* [19] if for any vertex $v \in V_i$, $i \in \{0, 1\}$, there is a Hamiltonian path of $G - v$[1] between any two vertices of V_{1-i}. A graph G is *pancyclic* if it contains cycles of every length from its *girth* (the length of a shortest cycle) to $|V(G)|$ inclusive. Since a bipartite graph does not contain odd cycles, the concept of bipancyclicity is proposed in [22]. A graph G is *bipancyclic* if it contains cycles of every even length from the girth of G to $|V(G)|$ if $|V(G)|$ is even, or to $|V(G)| - 1$ if $|V(G)|$ is odd. For graph-theoretic terminologies and notations are not mentioned here, readers may refer to [25].

An *n-dimensional hypercube* Q_n (n-cube for short) can be represented as an undirected graph such that $V(Q_n)$ consists of 2^n vertices which are labeled as binary strings of length n from $\underbrace{00\ldots0}_{n}$ to $\underbrace{11\ldots1}_{n}$. Each edge $e = (u, v) \in E(Q_n)$ connects two vertices u and v if and only if u and v differ in exactly one bit of their labels, i.e., $u = b_n b_{n-1} \ldots b_k \ldots b_1$ and $v = b_n b_{n-1} \ldots \bar{b}_k \ldots b_1$, where $\bar{b}_k$ is the *one's complement* of b_k, i.e., $\bar{b}_k = 1 - i$ iff $b_k = i$ for $i \in \{0, 1\}$. We call that e is an edge of *dimension k*. Clearly, each vertex connects to exactly n other vertices. In addition, there are 2^{n-1} edges in each dimension and $|E(Q_n)| = n \cdot 2^{n-1}$. Fig. 1 shows a 2-dimensional hypercube Q_2 and a 3-dimensional hypercube Q_3.

Let $x = x_n x_{n-1} \ldots x_1$ and $y = y_n y_{n-1} \ldots y_1$ be two n-bit binary strings; and let $y = x^{(k)}$, where $1 \leq k \leq n$, if $y_k = 1 - x_k$ and $y_i = x_i$ for all $i \neq k$ and $1 \leq i \leq n$. In addition, let $y = \bar{x}$ if $y_i = 1 - x_i$ for all $1 \leq i \leq n$. The *Hamming distance* $d_H(x, y)$ between two vertices x and y is the number of different bits in the corresponding strings of the vertices. The *Hamming weight* $hw(x)$ of x is the number of i's such that $x_i = 1$. Note that Q_n is a bipartite graph with two partite sets $\{x|\ hw(x)$ is odd$\}$ and $\{x|\ hw(x)$ is even$\}$. Let $d_{Q_n}(x, y)$ be the *distance* between two vertices x and y in graph Q_n. Clearly, $d_{Q_n}(x, y) = d_H(x, y)$.

An *n-dimensional folded hypercube* FQ_n can be constructed from an n-cube by adding an edge (also called *complementary edge*) to every pair of vertices that are the farthest apart, i.e., for a vertex whose address is $b = b_n b_{n-1} \ldots b_1$, it now has

[1] The graph obtained by deleting v from G.