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Combinatorial voter control in elections [☆,☆☆]

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ABSTRACT

Voter control problems model situations such as an external agent trying to affect the result of an election by adding voters, for example by convincing some voters to vote who would otherwise not attend the election. Traditionally, voters are added one at a time, with the goal of making a distinguished alternative win by adding a minimum number of voters. In this paper, we initiate the study of combinatorial variants of control by adding voters. In our setting, when we choose to add a voter v, we also have to add a whole bundle $\kappa(v)$ of voters associated with v. We study the computational complexity of this problem for two of the most basic voting rules, namely the Plurality rule and the Condorcet rule.

1. Introduction

We study the computational complexity of control by adding voters [2,31], investigating the case where the sets of voters that we can add have some combinatorial structure. The problem of election control by adding voters models situations where some agent (for example, a campaign manager of one of the alternatives) tries to ensure a given alternative's victory by convincing some undecided voters to vote. Traditionally, in this problem we are given a description of an election (that is, a set C of alternatives and a set V of voters who already decided to vote), and also a set W of undecided voters. For each voter in $V \cup W$ we assume that we know how this voter intends to vote, which is expressed as a linear order over the set C; while this assumption is somewhat unrealistic, it is a standard assumption within computational social choice, and we might have a good approximation of this knowledge from preelection polls. Our goal is to ensure that our preferred

☆ A preliminary short version of this work has been presented at the 39th International Symposium on Mathematical Foundations of Computer Science (MFCS 2014), Budapest, August 2014 [12]. In this long version we provide an additional fixed-parameter tractability result (Theorem 5) and all proofs that were omitted in the conference version.

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alternative p becomes a winner by convincing as few voters from W as possible to vote—provided that it is at all possible to ensure p's victory in this way.

Control by adding voters corresponds, for example, to situations where supporters of a given alternative make direct appeals to other supporters of the alternative to vote. For example, they may stress the importance of voting or help with the voting process by offering rides to the voting locations. Unfortunately, in its traditional phrasing, control by adding voters does not model larger-scale attempts at convincing people to vote. For example, a campaign manager might be interested in airing a TV advertisement that would motivate supporters of a given alternative to vote (though, of course, it might also motivate some of this alternative's enemies), or maybe launch viral campaigns, where friends convince their own friends to vote. It is clear that the sets of voters that we can add should have some sort of a combinatorial structure. For instance, a TV advertisement appeals to a particular group of voters and we can add all of them at the unit cost of airing the advertisement. A public speech in a given neighborhood will convince a particular group of people to vote at the unit cost of organizing the meeting. Convincing a person to vote will "for free" also convince her friends to vote.

The goal of our work is to formally define an appropriate computational problem modeling a combinatorial variant of control by adding voters and to study its computational complexity. We focus on the Plurality rule and the Condorcet rule, and we do so for the following reasons. First, these are the rules originally studied by Bartholdi et al. [2] in the first paper on the complexity of election control. Second, the Plurality rule is the most widely used rule in practice and the Condorcet rule models a large family of Condorcet-consistent rules. Third, the Plurality rule is one of the few rules for which the standard variant of control by adding voters is solvable in polynomial time [2]. For the Condorcet rule the problem is NP-hard in general [2], but becomes polynomial-time solvable if we assume that the elections have a particular structure (for example, if they are either single-peaked [24] or single-crossing [37]). For the case of single-peaked elections, in essence, all our hardness results for the Condorcet rule directly translate to all Condorcet-consistent voting rules, a large and important family of voting rules. We defer the formal details, definitions, and concrete results to the following sections. Instead, we state the high-level main messages of our work. Herein, we assume that adding an unregistered voter means adding a *bundle* (subset) of unregistered voters; in this way, it is easy to see that the standard variant of control by adding voters is a special case of the combinatorial variant (set the bundle of each unregistered voter to be a singleton consisting of this single voter):

1. Many typical variants of combinatorial control by adding voters are intractable, but there is also a rich landscape of tractable cases. For instance, with bundle sizes up to two, the problem is either fixed-parameter tractable with respect to the number k of bundles to add or already polynomial-time solvable when requiring the bundling function to be full-d (see Section 2 for the definition; informally, this means that only voters with roughly the same preference orders can be bundled together).
2. Assuming that voters have single-peaked preferences does not lower the complexity of the problem (even though it does so in many other election problems [7,14,24]). On the contrary, assuming single-crossing preferences does lower the complexity of the problem.

We believe that our setting of combinatorial control, and—more generally—of combinatorial problems that model manipulating elections, offers a very fertile ground for future research and we intend the current paper as an initial step.

Related work Bartholdi et al. [2] were the first to study the concept of election control by adding/deleting voters/alternatives in a given election. They considered the constructive variant of the problem, where the goal is to ensure a given alternative's victory (and we focus on this variant of the problem as well). The destructive variant, where the goal is to prevent someone from winning, was introduced by Hemaspaandra et al. [31]. These papers focused on the Plurality rule and the Condorcet rule (for the destructive case of Hemaspaandra et al. [31], also the Approval rule). Since then, many other researchers extended this study to a number of other rules and models [39,22,23,45,25,43].

We study our control problems using the tools and methods of parameterized complexity theory. Most frequently, parameterized complexity of control problems is studied with respect to the number of alternatives as the parameter [22,23,32]. The number of voters received far less attention as a parameter (for the case of control, the parameter appears, for example, in the works of Betzler and Uhlmann [3] and, very recently, of Chen et al. [13]; Brandt et al. [8] consider it in the context of winner determination). Several authors have also considered other parameters, such as the solution size (for example, the number of voters one can add). Papers focusing on this parameter include, for example, those of Liu et al. [36], Liu and Zhu [35], and Erdélyi et al. [20].

Some of our results regard the complexity of election control for the case where the voters' preference orders are either single-peaked [5] or single-crossing [41,44] (intuitively, both these domain restrictions model cases where there is a linear spectrum of opinions; single-peakedness assumes that it is the alternatives that are ordered from one extreme to the other within this spectrum, for example, the left-to-right political spectrum, while single-crossingness assumes that the order is over the voters and their opinions). For both types of domain restrictions there are algorithms that can recognize elections with a given property (see, for example, the works of Bartholdi and Trick [1] and Escoffier et al. [21] for the single-peaked domain and those of Elkind et al. [18] and Bredereck et al. [9] for the single-crossing domain). The complexity of control for single-peaked elections was studied by Faliszewski et al. [24] and was continued, for example, by Brandt et al. [7] and Faliszewski et al. [26]. The case of control in single-crossing elections was considered by Magiera and Faliszewski [37]. Gen-