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Precise contact motion planning for deformable planar curved shapes ☆

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HIGHLIGHTS

- We present an efficient motion planning algorithm for a planar deformable robot.
- We employ K-contact motion analysis to reduce the degrees of freedom of the robot.
- Our algorithm efficiently finds a feasible path via a graph searching algorithm.

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ABSTRACT

We present a precise contact motion planning algorithm for a deformable robot in a planar environment with stationary obstacles. The robot and obstacles are both represented with C^1-continuous implicit or parametric curves. The robot is changing its shape using a single degree of freedom (via a one-parameter family of deformable curves). In order to reduce the dimensionality of the configuration space, geometrically constrained yet collision free contact motions are sought, that have $K(=2,3)$ simultaneous tangential contact points between the robot and the obstacles. The K-contact motion analysis effectively reduces the degrees of freedom of the robot, which enables a more efficient motion planning. The geometric conditions for the K-contact motions can be formulated as a system of non-linear polynomial equations, which can be solved precisely using a multivariate equation solver. The solutions for K-contact motions are represented as curves in a 4-dimensional (x, y, θ, t) space, where x, y, θ are the degrees of freedom of the rigid motion and t is the deformation's parameter. Using the graph structure of the solution curves for the K-contact motions, our algorithm efficiently finds a feasible path connecting two configurations via a graph searching algorithm, whenever available. We demonstrate the effectiveness of the proposed approach using several examples.

1. Introduction

The problem of collision-avoidance in robot motion planning has been an active research area over the last several decades. Remarkable progress has been achieved in numerous practical applications. However, relatively few results were introduced for the case of deformable robots, even though potentially many applications can benefit from the added degrees of freedom. The deformation ability of the robot provides more flexibility in the motion planning and enables the successful navigation of more challenging tasks that cannot be accomplished by rigid robots. Actual designs of deformable robots already exist, i.e. [1]. On the other hand, more degrees of freedom in the robot exponentially increase the complexity of motion planning algorithms. As a consequence, it is very difficult to find feasible motions in a reasonable amount of time.

The geometry of deformable robots and their environments are often designed with Non-Uniform Rational B-spline (NURBS) curves and surfaces, which is the de facto standard representation for industrial objects. Yet, the majority of contemporary algorithms for motion planning first tessellate the freeform NURBS curves and surfaces, as they can handle only piecewise linear discrete objects. The errors caused by the polygonal approximation are difficult to control. Moreover, it is non-trivial to precisely manage the collision detections (in particular when dealing with contact motions) using these polygonal approximations. Thus, it is highly desirable to

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directly process the NURBS curves and surfaces for applications requiring high accuracy.

In this paper, we consider the precise (up to machine precision) contact motion planning for a deformable planar parametric or implicit freeform C^1-continuous robot $\Phi_t(C(u))$, with respect to a C^1-continuous stationary (set of) parametric obstacle(s) $D(v)$, in the plane. The transformation Φ_t represents a one-parameter smooth freeform deformation of the C^1-continuous robot $C(u)$ and we assume that Φ_t is pre-defined algebraically. The deformable robot $\Phi_t(C(u))$ has two rigid motion degrees of freedom in translation, (x, y), and one rigid motion degree of freedom in rotation, θ. Finally, the last degree of freedom t provides the shape control over the robot's deformation function Φ_t. The configuration space (C-space) is thus a 4-dimensional space, in (x, y, θ, t).

A naive approach for planning the motion of such a deformable robot is to compute the entire boundary of the C-space's obstacle which can be represented as an implicit 3-manifold, $f(x, y, \theta, t) = 0$, and use this 3-manifold for the motion planning. The optimal motion path may then be computed by considering all possible motion paths over the entire boundary of the obstacle's C-space. Clearly, computing and even representing the entire 3-manifold solution is expected to be highly challenging. It is indeed inefficient and in fact unnecessary to compute the entire boundary of the C-space, because the robot typically follows a univariate motion path and thus only a small portion of the C-space is used for the motion planning.

Therefore, instead of computing the entire C-space, we focus on analyzing contact motions that satisfy additional geometric constraints so that the dimension of the computed solution can be significantly reduced. Toward this end, we seek the collision free motion of a (deformable) robot $\Phi_t(C(u))$ while it maintains multiple tangential contacts, typically two or three, with obstacles $D(v)$. We denote such motions $K(=2, 3)$-contact motions. The reasons for using K-contact motions, in the motion planning, are twofold:

1. We can now effectively reduce the degrees of freedom in the contact motion analysis. Now, every K-contact motion will be represented as a curve in a 4-dimensional C-space. As a result, the entire set of K-contact motions forms a graph structure in the (x, y, θ, t) space and the motion planning can be addressed via well-known graph searching algorithms.
2. A K-contact motion analysis often provides a good solution for narrow passage problems, because there is a high probability for the robot to have multiple contact points in such narrow passages. Fig. 1 shows an example of a deformable robot with K-contact motion that follows a narrow passage.

The rest of this paper is organized as follows. In Section 2, we briefly review previous related work. Section 3 describes the different ways one can prescribe the deformation of the robot, algebraically. Section 4 introduces the algebraic conditions for the K-contact motion between $\Phi_t(C(u))$ and $D(v)$. Section 5 addresses the construction of the K-contact motion graph and the motion planning algorithm using this K-contact graph. Several experimental results are reported in Section 6 and the paper is finally concluded with discussions on future work in Section 7.

2. Related work

Algorithms for motion planning of deformable robots are typically based on the probabilistic road-map (PRM) approach [2]. A PRM planner samples random points in the C-space and generates an approximated C-space graph by connecting adjacent sample points. Then, the planner finds a feasible motion by connecting these points on the graph via a graph searching algorithm. The

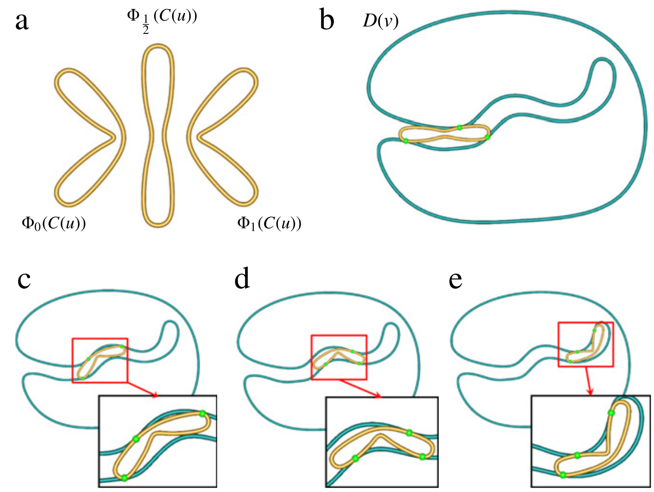

Fig. 1. K-contact motion planning for a deformable robot: (a) a deformable robot $\Phi_t(C(u))$ and the shape of $\Phi_t(C(u))$ for $t = 0, \frac{1}{2}, 1$. (b) The robot with constant deformation parameter ($t = \frac{1}{2}$) cannot go deeper into obstacle $D(v)$ without gouging at the 3-contact configuration. (c)–(e) A 3-contact motion analysis between robot $\Phi_t(C(u))$ and obstacle $D(v)$ enlarges the accessible region of the, now deformable, robot.

performance and quality of PRM based algorithms is heavily dependent on the sampling strategy. Guibas et al. [3] proposed an efficient motion planning algorithm for flexible objects. By deforming the robot as closely as possible to the medial axis of the workspace, the algorithm successfully finds critical deformations and effectively reduces the deformation space. However, the medial axis computation for 3D objects is computationally expensive and fitting the object to the 3D medial axis is non-trivial. Bayazit et al. [4] sampled configurations that might cause inter-penetration into the obstacles and then generated a collision-free path by locally deforming the robot. The deformation of the robot is employed only for preventing the inter-penetration of the robot and thus limits the exploration of the deformation space. Gayle et al. [5] developed a practical algorithm for motion planning of deformable robots in complex environments, which can take into account geometric and physical constraints. The deformation of the robot should satisfy some imposed constraints that are formulated as an energy minimization problem and be solved via an optimization algorithm. This optimization can be inefficient if the geometry of the robot and the environments are not similar. Mahoney et al. [6] tackled the problem of motion planning of deformable robot by reducing the dimension of the deformation space via principal component analysis over the deformation space. The entire deformation space is reduced to the subspace spanned by a small number of basis elements and the degrees of freedom in C-space are significantly reduced. The problem is that, in many cases, the reduced deformation space suffers from narrow passages of the obstacles.

The above results focus on polygonal representation and typically produce approximate solutions. The previous work on precise motion planning for NURBS curves and surfaces is limited. Bajaj and Kim [7–9] considered the generation of C-space obstacles for translational motions of rigid algebraic curves and surfaces. For the case of a rigid planar freeform shapes moving (with translation only) among similar static freeform curves in the plane, Lee et al. [10] presented a high-precision algorithm that can approximate the boundary of the obstacle's C-space using B-spline planar curves. Holleman et al. [11] and Lamiraux et al. [12] applied a PRM planner for path planning of flexible surface patches modeled with low degree Bézier surfaces. Holleman et al. [11] enforced a desired deformation of the surface by formulating the