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A B-spline based framework for volumetric object modeling[☆]

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ABSTRACT

With the recent development of Iso-geometric Analysis (IGA) (Cottrell et al., 2009) and advanced manufacturing technologies employing heterogeneous materials, such as additive manufacturing (AM) of functionally graded material, there is a growing emerging need for a full volumetric representation of 3D objects, that prescribes the interior of the object in addition to its boundaries. In this paper, we propose a volumetric representation (V-rep) for geometric modeling that is based on trimmed B-spline trivariates and introduce its supporting volumetric modeling framework. The framework includes various volumetric model (V-model) construction methods from basic non-singular volumetric primitives to high level constructors, as well as Boolean operations' support for V-models. A V-model is decomposed into and defined by a complex of volumetric cells (V-cells), each of which can also represent a variety of additional varying fields over it, and hence over the entire V-model. With these capabilities, the proposed framework is able of supporting volumetric IGA needs as well as represent and manage heterogeneous materials for AM. Further, this framework is also a seamless extension to existing boundary representations (B-reps) common in all contemporary geometric modeling systems, and allows a simple migration of existing B-rep data, tools and algorithms. Examples of volumetric models constructed using the proposed framework are presented.

1. Introduction

In geometric modeling (GM), 3D objects are mainly represented by their boundaries [1]. Typically, these boundaries are represented as a set of tensor product (trimmed) surfaces. These surfaces define the 2-manifold boundaries of the object and hence delineate its volume. Until recently, full volumetric representation of 3D objects has not been in high demand in the GM and engineering communities. However, with the development of advanced manufacturing technologies employing heterogeneous materials such as additive manufacturing [2] (AM, also known as 3D printing) using functionally graded materials [3], methods for representing materials in the entire volume of the object are in active demand and research. The object's description should include its geometry as well as other relevant internal volumetric data sets, like material's properties, such as stresses or conductivity fields, and also boundary conditions, like pressure. Hereafter, we refer to these non-geometric data sets and fields as *attributes*. These volumetric capabilities are simply lacking in contemporary GM systems.

3D volumetric modeling is in demand also in micro-scale bone scaffold design, where patient's specific porous structures are designed, in order to replace unhealthy or diseased tissue [4], and in the design of porous material in general (i.e. topological optimizations [5]).

Traditional finite elements analysis (FEA) processes require the conversion of 3D B-rep data, such as mechanical parts, to a representation in which the physical simulation can be employed. Grids or meshes, based on piecewise linear approximating primitives like triangles, tetrahedra, quadrilaterals and hexahedra are frequently used representations for analysis [6]. The drawback of such a discrete representation is the lack of numerical stability on one hand, and accuracy on the other—the generated models are merely piecewise linear discrete approximations of the real freeform models [7]. [7] reports that the process of generating a discrete approximated mesh for analysis from a given 3D B-rep CAD model is the key, most time consuming, step in finite element analysis (FEA). It is estimated to consume about 80% of the overall design and analysis process in the automotive, aerospace and ship industry. Thus, employing a single geometric representation for both the design and the analysis, throughout the modeling cycle, has major potential advantages.

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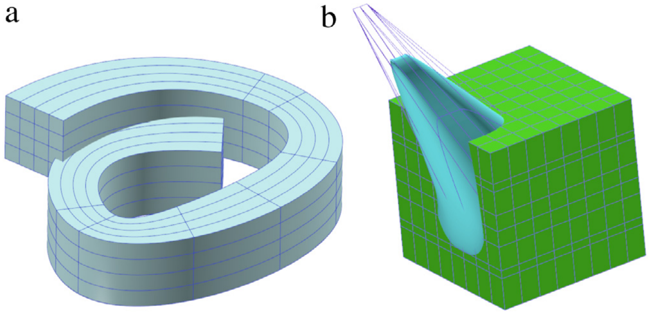

Fig. 1. (a) Shows a non-regular (singular) self-intersecting B-spline trivariate whereas (b) shows a non-regular cuboid B-spline trivariate where three internal control points are moved up and outside the cuboid, resulting in a zone of negative Jacobian (in cyan). Note the boundary of this trivariate no longer consists of only the six boundary surfaces. See also [13]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Isogeometric analysis (IGA) postulates the use of the same tensor product B-spline [1] representation used for representing the geometry in the physical analysis as well, striving for a tighter bond between GM and Analysis. The end user can work in the same B-spline representation in the design and analysis stages, avoiding both the need to generate a finite element intermediate mesh toward the analysis and the need for a complex feedback of the analysis results back to the B-spline based geometric model. However, it is necessary to have a B-spline based representation that is suitable for analysis. Over the last decade, several studies have been conducted to utilize IGA for different physical analysis problems, i.e. [8–11], showing improved accuracy and robustness over traditional finite element methods [12]. So far, these studies are, for the most part, designed for the 2D case, (and to some extent trimmed surfaces), due to the lack of suitable geometrical representation and tools for handling volumetric objects.

In this work, we introduce a framework and techniques for representing and managing freeform volumetric objects, having a full representation of the boundary as well as the interior volume of the model, taking into account both accuracy and efficiency. In this work, we only deal with volumetric models (V-model) that are open sets and regular. That is, a V-model is a regular 3-manifold geometry and the boundary (closure) of the V-model is a closed regular 2-manifold. A regular 3-manifold (2-manifold) does not self-intersect, and has a vanishing Jacobian in no place. See Fig. 1 for examples of a non-regular (singular) trivariate. As stated, in this work, we exclude such singular cases.

The main contributions of this V-rep framework are:

1. A data structure for accurately representing a general freeform V-model, its geometry as well as scalar, vector, and tensor fields in its interior and/or over its boundaries.
2. Constructors of basic volumetric primitives as a complex of non-singular B-spline trivariates, such as a cylinder, a torus and a sphere and also more advanced constructors such as ruled volumes and volumes of revolution.
3. Algorithms for Boolean operations over V-models, that are based on trimmed B-spline trivariate vector functions.
4. Geometric support tools for precise management and analysis.
5. Almost seamless conversion from B-rep geometry.

The rest of this document is organized as follows. Section 2 discusses related work. In Section 3, we describe our V-model representation (V-rep). In Section 4, we present the volumetric Boolean operations between V-models, for creating arbitrarily complex V-models. In Section 5, we portray unique geometric tools that are supported in the framework and enable precise analysis over V-models. In Section 6, we present some results of constructions of V-models using the proposed V-rep modeling framework, and finally, in Section 7, we conclude and discuss future planned research.

2. Related work

The most common geometric representation of 3D objects is the boundary representation (B-rep), where the object is delineated by its boundary surface(s). There are many methods to model the boundary surface(s), and the two most common basic building blocks for B-reps are:

- Linear primitives such as triangles, quads or general polygons, and
- Trimmed spline surfaces, as a set of piecewise polynomial or rational functions, over some parametric spaces.

These methods are commonly used in contemporary GM systems for about half a century, with little change. Until recently, representing the inner volume of the object has been of little interest. However, in recent years, there has been a growing need for modeling the interior volume of geometric objects, for example in engineering, medicine and manufacturing. AM with graded materials is already a proven technology and IGA requires an appropriate representation for various prescribed and/or computed fields in the interior of an object.

The voxel based volume representation is common in medical applications and is simple to manipulate. However, voxel based approaches suffer from lack of accuracy and huge data sizes. For example, a CT scan from a typical device can generate a volume of 512 voxels in each dimension (a 2 mm accuracy for an object of one meter wide), with each voxel represented in typically 16 bit. Such a volume requires around 256 MB of storage space. In contrast, the accuracy offered by current subtractive manufacturing (SM) technologies (i.e. CNC) is in the orders of microns and tens of microns, which is 2–3 orders of magnitude better, and AM is expected to follow these accuracies in the near future. There is little hope that voxel based representation can provide such accuracies, considering the amount of expected storage space.

There have been several relevant studies on spline based volumetric representations. Basic constructors of tensor product B-spline trivariates have been investigated in [14]. In [13], algorithms to derive the boundaries of tensor product trivariates with singularities (vanishing and/or varying-sign Jacobians) are proposed. Aigner et al. [15] proposed an algorithm for calculating a tensor product trivariate from boundary conditions and guiding curves. Their method is proposed to handle only swept volume structures. Liu et al. [16] use Boolean operation of volumetric cylinders and cubes with hierarchical octrees to extract T-spline trivariates from boundary triangulated surfaces. However, [16] cannot handle more complex shapes, even as simple as cones and tetrahedron.

Kumar et al. [17–19] provide a good summary on the various mathematical representations for models found in the literature. They proposed a framework for segmenting the object into cells by using constructive solid geometry (CSG) operations, where each cell describes both the geometry and material properties. However, their method uses a limited set of basic shapes (and CSG operations) like spheres and cubes, which makes their method too restrictive to handle complex general freeform objects.

Martin et al. [20,21] modeled the attributes as separate trivariate volumes that are not coupled to the geometry, but share the same parametric domain. Their method supports only complete (non-trimmed) tensor product trivariates.

Biswas et al. [22] propose a high level abstract model for the representation of heterogeneous objects, that are composed of geometry and continuously varying materials. Their extension toward heterogeneous materials is based on a distance function from interior or boundary geometric curves or points defining the material attributes, called features positions. Chen et al. [23] propose a framework for representing and optimizing volumetric