Computer-Aided Design 78 (2016) 71–82

Contents lists available at ScienceDirect

Computer-Aided Design

journal homepage: www.elsevier.com/locate/cad

Homogenization of material properties in additively manufactured structures[☆]

Xingchen Liu, Vadim Shapiro [*]

Spatial Automation Laboratory, University of Wisconsin - Madison, United States

ARTICLE INFO

Keywords:
Additive manufacturing
Fused deposition modeling
Printed model
Heterogeneous material
Homogenization

ABSTRACT

Additive manufacturing transforms material into three-dimensional parts incrementally, layer by layer or path by path. Subject to the build direction and machine resolution, an additively manufactured part deviates from its design model in terms of both geometry and mechanical performance. In particular, the material inside the fabricated part often exhibits spatially varying material distribution (heterogeneity) and direction dependent behavior (anisotropy), indicating that the design model is no longer a suitable surrogate to consistently estimate the mechanical performance of the printed component.

We propose a new two-stage approach to modeling and estimating effective elastic properties of parts fabricated by fused deposition modeling (FDM) process. First, we construct an implicit representation of an effective mesoscale geometry–material model of the printed structure that captures the details of the particular process and published material information. This representation of mesoscale geometry and material of the printed structure is then homogenized at macro scale through a solution of an integral equation formulated using Green's function. We show that the integral equation can be converted into a system of linear equations that is symmetric and positive definite and can be solved efficiently using conjugate gradient method and Fourier transform. The computed homogenized properties are validated by both finite element method and experiment results. The proposed two-stage approach can be used to estimate other effective material properties in a variety of additive manufacturing processes, whenever a similar effective mesoscale geometry–material model can be constructed.

1. Introduction

1.1. Motivation and goals

Additive manufacturing (AM) represents a spectrum of technologies producing 3D parts incrementally, layer by layer or path by path. This distinctive feature gives AM numerous advantages over the traditional manufacturing techniques, such as the ability to fabricate parts with complex shapes and internal structures without a significant increase in cost or turnaround time. In many cases, a complex heterogeneous structure with less material may be both cheaper and faster to manufacture than a part with a simpler geometry and homogeneous material, such as a solid cube. This phenomenon is sometimes referred to the "complexity paradox".

Similarly to other manufacturing methods, the quality of additively manufactured parts is subject to the process limitations and machine imprecision. Various differences between designed and manufactured parts have been studied in the past, in terms of surface roughness [1,2], dimensional accuracy [3,4], and other manufacturability criteria [5,6]. Experiments have also been conducted to estimate the material properties of the printed part. The test specimens are printed in shapes per material testing standards (e.g. ASTM D3039 [7]) with material deposition paths (roads) aligned along the axial, transverse or cross directions [8,9].

In contrast to many traditional manufacturing processes, the material undergoes a fundamental phase transformation during the AM process, changing not only its geometry but also its mechanical properties. Processing plan and parameters in AM also play a more significant role in the final performance of the part—the same nominal part geometry manufactured with two different set of process plans will generally result in parts with very different properties. As a result, all AM processes lead to a heterogeneous and anisotropic distribution of material properties in the interior of

---

[☆] This paper has been recommended for acceptance by Scott Schaefer and Charlie C.L. Wang.

[*] Corresponding author.
E-mail addresses: xingchen@wisc.edu (X. Liu), vshapiro@engr.wisc.edu (V. Shapiro).

http://dx.doi.org/10.1016/j.cad.2016.05.017

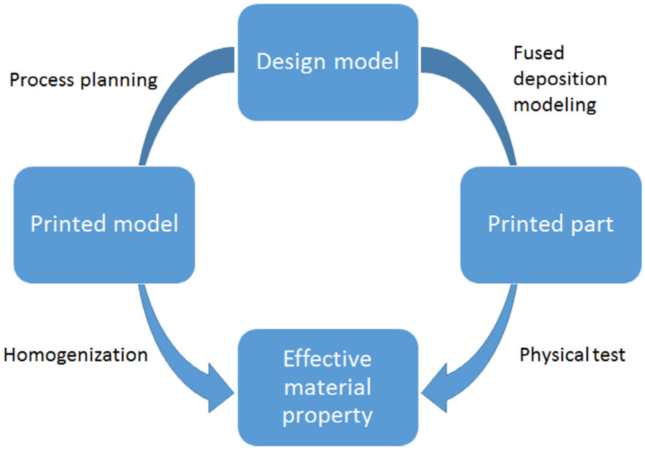

Fig. 1. To model and estimate the effective material properties of parts made by FDM, the proposed approach includes two stages shown in the left branch: modeling of printed material and numerical homogenization. Direct simulation of the design model is likely to yield poor results due to the various inconsistencies between the design model and the printed part. The proposed method is verified with the physical testing result in [10] on the right branch in Section 5.

the fabricated part, which is usually not represented and accounted for in the part's design model. In other words, the design model is no longer a suitable surrogate for the fabricated part. The accuracy of downstream applications, such as structural analysis, relies on the ability to model not only the manufactured part's geometry but also its material's mechanical properties.

The material properties may be estimated at least three distinct scales: material phase changes take place at the micro scale, allowing planning material deposition at the (meso) scale of layers and paths, which are fused together to give effective mechanical properties of the manufactured part's (macro) scale. As AM is rapidly evolving from a technology to prototype products in the conceptual design stage into a manufacturing process for the end-use load-bearing functional components, it is imperative to develop a computational infrastructure that allows mechanical analysis to be performed directly on the manufactured part. *Estimating such effective properties at the scale of the part's geometry is the goal of this paper.*

1.2. Contributions and outline

Specifically, we propose a new approach to modeling and estimating the effective (macroscopic) material properties in the interior of the parts produced by the Fused Deposition Modeling (FDM) process using homogenization. Informally, homogenization replaces the known detailed geometry and multi-phase material properties at a finer scale by simpler 'effective' geometric domain and single-phase (solid) material properties at a coarser scale. The effective domain is usually a cuboid, and the effective material properties are estimated from average stress and strain relationship over the cuboid. Homogenization is challenging for AM parts for two reasons: (1) geometry and material properties may not be known at the micro and mesoscales; and (2) homogenization requires significant computational resources.

The proposed approach deals with the two challenges in two stages (Fig. 1): a modeling stage that generates a representation of (mesoscale) geometry and anisotropy of the material deposition in the interior of the part, followed by an efficient analysis stage homogenizing the generated 3D-printed structure for its effective material properties. The concept of homogenization is extensively used in both two stages.

In the modeling stage, given a manufacturing process plan in the form of G-code that describes the printer's toolpath, we construct an effective geometry–material model to represent the heterogeneous distribution and anisotropic material properties of the FDM printed structures. The construction combines an analytic model of geometry with experimentally measured material properties that are linked together by homogenization *assumed* in the measurement procedure. We also describe an implicit representation of this mesoscale effective geometry–material model that supports efficient queries and can be evaluated on demand for further processing. This is the first contribution of the paper detailed in Section 3.

In the second stage, described in Section 4, the mesoscale geometry–material model of the printed structure is homogenized to obtain the effective (macro-scale) material elasticity tensor. We adopt Green's function method that is often used in the studies of random heterogeneous materials. We convert the formulated integral equation into a system of linear equations and show that the linear system is symmetric and positive definite with properly chosen reference material. This is our second contribution, which gives a formal basis for using efficient homogenization techniques. The symmetric and positive definite linear system is solved by the conjugate gradient (CG) method. The matrix–vector multiplication required by CG is equivalent to the convolution between Green's operator and the vector of polarized stresses, and thus can be evaluated efficiently through Fourier transform.

To validate our results, in Section 5, we show that the computed results are consistent with those obtained by a more traditional (but an order of magnitude slower) homogenization method based on finite element method. We also apply the complete two-stage modeling-homogenization approach to models of printed material samples and show that predicted effective material properties are in agreement with the physical tests performed on the same structures.

2. Background and related work

2.1. Fused deposition modeling

FDM is a widely used AM process that produces parts with significant material anisotropy and heterogeneity that cannot be neglected. Parts built by FDM differ noticeably from their design models due to many factors, including stair—stepping on the surface of the part, the rounding of sharp corners, air gaps and the use of infill patterns to save the printing material and printing time, impacting the mechanical performance of the part.

To manufacture by FDM, the design (solid) model is first converted to a stereolithography (STL) file, which represents the solid part by a triangle tessellation of its boundary. The STL model is subsequently sent to a process planning software (e.g. Slic3r [11]) that generates the printer's head toolpath together with the printing process specifications, such as build direction, nozzle diameter, and infill percentage. As the printer's head moves, a molten filament is extruded through a heated nozzle. For each layer, the nozzle moves following a piece-wise linear path horizontally. The material extruded along each line segment is commonly referred to as a 'road'. After each deposition, the road solidifies and bonds with adjacent roads in both current and previously deposited layers. After the whole layer is deposited, either the nozzle or the printing plate shifts vertically to print the next layer.

Much research has been dedicated to model the geometric differences between the design model and printed part; a comprehensive survey is beyond the scope of this paper. For example, elliptical model of the road cross-section is proposed in [12] to analyze surface roughness distribution according to changes in the angle between the surface and the build direction. Manufacturability of the designed part is examined in [5]