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Modeling and analysis of origami structures with smooth folds[☆]

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ABSTRACT

Origami has the potential to impact numerous areas of design and manufacturing. Modeling and analysis of origami structures allow for the understanding of their behavior and the development of computational tools for their design. Most available origami models are limited to the idealization of folds as creases of zeroth-order geometric continuity, which is not proper for origami structures having non-negligible fold thickness or with maximum curvature at the folds restricted by material limitations. Structural analysis of origami sheets having creased folds requires further idealizations of the fold mechanical response such as the representation of the folds as torsional springs. In view of this, a novel model analogous to that for rigid origami is presented in this work for origami structures having folds of non-zero surface area that exhibit higher-order geometric continuity (termed *smooth folds*). This origami model allows for a proper structural analysis of origami sheets using plate or shell representations for the folds. The shape formulation of the smooth folds and the kinematic constraints on their associated shape variables are presented. Modeling of origami structures with smooth folds exhibiting elastic behavior is performed by determining the configuration of the structure that minimizes its total potential energy subject to the derived kinematic constraints. The presented results show that the structural response determined using the proposed model is in good agreement with both experiments and higher-fidelity finite element analyses.

1. Introduction

Traditionally, in *origami* a goal shape is achieved exclusively by *folding* an initially planar sheet of negligible thickness. In this context, a *fold* is any deformation of the sheet such that the in-surface distance between any two points in the sheet is invariant and self-intersection does not occur [1]. Over the past four decades, there has been an increasing attention from the mathematics, architecture, science, and engineering communities given to theoretical models and computational design tools for origami [2,3]. Engineering advantages of origami-inspired structures include compact storage/deployment capability [4], the potential for reconfigurability [5], and a reduction in manufacturing complexity [6], among others [7,8].

Rigid origami is the special case of origami for which the planar faces bounded by the folds and the sheet boundary undergo only rigid deformations, i.e. these faces are neither bent nor stretched [10]. An example of rigid origami is shown in Fig. 2(a). Rigid origami has been studied by several researchers in the past and the subject remains active [11]. For example, it has been recently utilized for the design of pop-up mechanisms [12,13] and deployable structures [14,15].

Mathematical modeling and simulation of rigid origami structures allows for the understanding of their behavior and the development of computational tools for their design. Various approaches have been utilized for the simulation of rigid origami [10]. For example, truss representations [16] have been considered where the faces of the sheet are triangulated, each fold or boundary edge end-point is represented by a truss joint, and each fold and boundary edge is represented by a truss member. Configurations in which the truss joint displacements do not cause elongations on the truss members [17] represent valid rigid origami configurations.

Alternatively, Belcastro and Hull [18,19] have proposed a model for rigid origami in which the deformation caused by folding is represented using affine transformations. This model provides constraints on the fold angles allowing for valid rigid origami configurations as well as mappings between unfolded and folded

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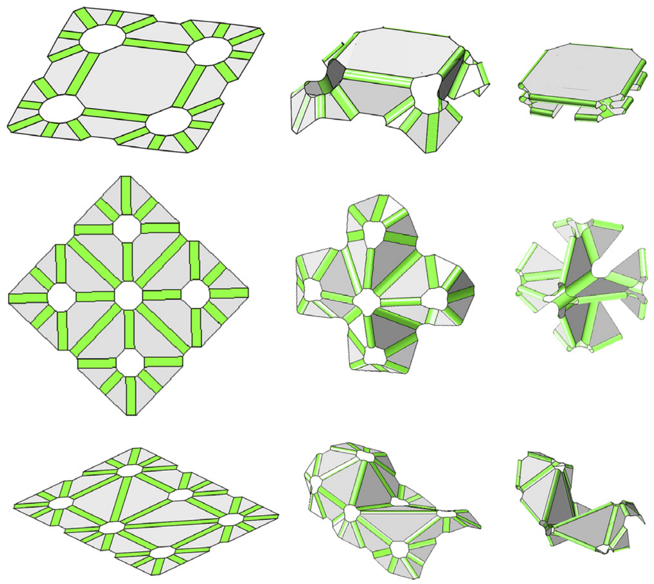

Fig. 1. Examples of valid configurations for origami structures with smooth folds. These examples are based on *kinematic* simulation [9]. The model presented in this paper extends beyond kinematics by considering *mechanical equilibrium* and *material constitutive behavior* and allows for a thorough physics-based analysis of origami structures with smooth folds.

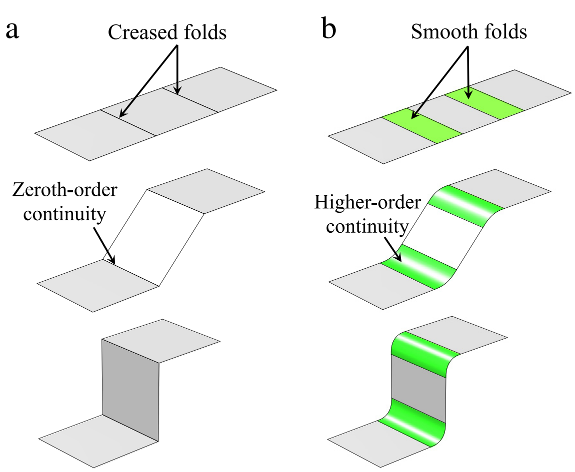

Fig. 2. (a) A sheet with creased folds of zeroth-order geometric continuity (G^0). (b) A sheet with *smooth folds* of non-zero surface area and higher-order geometric continuity (G^n).

configurations. Such a model provided the theoretical basis for origami simulation tools such as the *Rigid Origami Simulator* [20] and *Freeform Origami* [21], both developed by Tachi (see Refs. [10, 22]).

Most origami modeling approaches and design tools to date are based on the assumption of *creased folds* (see Fig. 2(a)). These folds are straight line segments[1] that divide the sheet into faces such that, upon folding, only zeroth-order geometric continuity (G^0) is maintained between faces (i.e. the sheet tangent plane may be discontinuous at these folds). This idealization of physically folded structures has been useful in the analysis and design of many origami-inspired applications throughout the years [15,26, 27]. However, such an idealization may not be appropriate for structures having non-negligible fold thickness or with maximum curvature at the folds restricted by material limitations (e.g. [28–31]). In these cases, the folds are not accurately represented as creases but rather as *bent* sheet regions exhibiting higher-order geometric continuity (as shown in Figs. 1 and 2(b)). Such regions will be referred to in this work as *smooth folds*.

Modeling of bent and creased surfaces has been performed using collections of planar, cylindrical, conical, and other ruled surface subdomains [32,33]. For example, Hwang and Yoon modeled three-dimensional developable surfaces via bending operations analogous to wrapping regions of an initially planar surface onto cylindrical and conical sections [33]. Zhu and coworkers developed a method for analyzing surfaces under creased and bent folds [34]. Their method allowed for the superposition of folds with arbitrary sharpness and fold angle that collectively dictated the ultimate shape of the analyzed surface. None of the aforementioned works [32–34] has addressed constraints on the fold angles or fold pattern that are required to preserve rigidly deformed sheet regions as are formed in analogous rigid origami models, which are critical when fold intersections are present in the sheet.

Besides purely kinematic models, there have been efforts to model the structural response of origami sheets. Schenk and Guest [16] proposed a model for origami structures with elastic creased folds based on truss representations. Their model introduces torsional spring behavior at the creases. Tachi used a similar approach to model the elastic behavior of sheets with creased folds by also idealizing the folds as torsional springs and solving equations of mechanical equilibrium under constraints assuring that no fold line or boundary edge is elongated [35].

Representing folds as creases having torsional spring behavior may not be suitable for structures of significant thickness or comprised of materials that do not exhibit large enough strains to approximate creases (e.g. metal or active materials-based sheets). In these cases, the folds are properly modeled using plate or shell representations. A novel model for the structural (elastic) response of rigid origami structures having *smooth folds* is presented in this work. The higher-order continuity of smooth folds is required for their analysis using plate and shell representations. The presented kinematics provide constraints on the fold shape variables required for valid origami configurations. Such kinematic constraints are considered together with equations of mechanical equilibrium in order to model the elastic behavior origami sheets with smooth folds.

The outline of the paper is as follows: the theoretical preliminaries and the key kinematic equations for origami with smooth folds are presented in Sections 2 and 3, respectively. A model for origami sheets having elastic smooth folds is described in Section 4. A numerical approach used to implement this model is presented in Section 5. Details on the finite element and experimental analyses used to numerically and experimentally validate the proposed model are briefly presented in Sections 6 and 7, respectively. Section 8 presents simulation results of origami sheets using the proposed model, and Section 9 provides a summarizing discussion and concludes the paper.

2. Theoretical preliminaries

The origami modeling approach adopted and extended in this work is based on the model presented in Refs. [18,19]. The studied continuum body denoted as the *sheet* and the shape variables associated with the smooth folds in the sheet (e.g. *fold angles*) are first described. The layout of the smooth folds (i.e. the *fold pattern*) is then determined by *vertices* (start-points and end-points of the line segments coincident with the smooth

[1] Curved creased folds are also feasible in origami (information on curved creases is provided in Refs. [23–25]); however, the focus of this work is on rigid origami-based structures for which curved folds are not allowed since their folding induces bending of the faces connected to such folds [24].