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Refinable C^1 spline elements for irregular quad layout

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ABSTRACT

Building on a result of U. Reif on removable singularities, we construct C^1 bi-3 splines that may include irregular points where less or more than four tensor-product patches meet. The resulting space complements PHT splines, is refinable and the refined spaces are nested, preserving for example surfaces constructed from the splines. As in the regular case, each quadrilateral has four degrees of freedom, each associated with one spline and the splines are linearly independent. Examples of use for surface construction and isogeometric analysis are provided.

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1. Introduction

Geometrically continuous G^k spline complexes and generalized subdivision surfaces are the two most popular families of constructions for filling multi-sided holes in a regular tensor-product spline lattice. Although many properties of subdivision surfaces can be computed by spectral analysis, their representation as an infinite sequence of ever smaller smoothly-connected surface rings complicates their inclusion into the existing industrial design infrastructure and their analysis computing integrals. By contrast, the finitely many patches of a G^k spline complex are typically more convenient and fit well into the CAD pipeline. However, refining a G^k spline complex requires careful book keeping. For, what used to be an edge where patches join with G^k continuity is now split into two. To represent the same spline complex, the pieces further away from the multi-sided hole have to remember to join using G^k rules rather than the regular C^k rules that the immediate regular neighborhood seems to prescribe. If we ignore the book keeping, we can refine G^k spline complexes, but the resulting spaces are not nested. That is, an initial surface or function will typically not have an exact representation in refined form. By contrast, subdivision functions yield nested spaces by construction.

Can we combine, at multi-sided configurations, a finite representation with simple nested refinability? After developing a solution, we realized that our solution was rather similar to work already published in U. Reif's Ph.D. thesis (1993, 1997). Reif proposed to project bi-cubic C^1 splines into a subspace that, despite being singular at the central point of the n-sided cap, guarantees tangent continuity at the central, irregular point and C^1 continuity everywhere else. However, there were two shortcomings: poor shape and a loss of degrees of freedom near the multi-sided configurations. Inconveniently, the projected space has fewer degrees of freedom near the irregularity than in the surrounding regular spline regions; and these degrees of freedom cannot be symmetrically distributed as proper control points. Our variant of Reif's construction applies one localized 2×2 split (see Fig. 1a) so that the resulting degrees of freedom are uniformly distributed and so that

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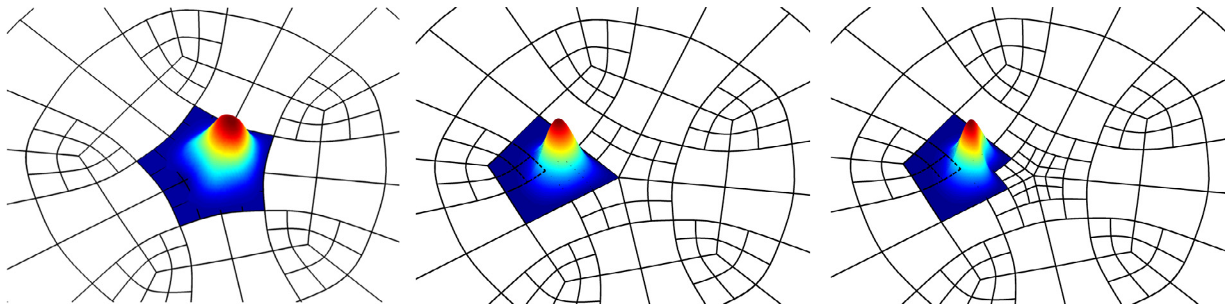

(a) Function f_{11} of a near-center control point c_{11}. (b) Function f_{22} of an off-center control point c_{22}. (c) PHT refinement at an $n = 5$ irregularity.

Fig. 1. Piecewise bi-cubic refinable C^1 basis functions f_{ij}. On quadrilaterals adjacent to irregular points, the basis functions of the spline space consist of 2×2 C^1-connected polynomial pieces. Surface and analysis space consist of linear combinations $\sum c_{ij} f_{ij}$ with control points c_{ij}.

▷ regardless of the valences of its vertices, each quadrilateral of the input (un-split) quad mesh is associated with four degrees of freedom (four basis functions).

In the regular case, where all the vertices of a quad have valence four, these four degrees of freedom are the B-spline control points of bicubic splines with double knots. Figs. 1a, b show basis functions near irregular points and the local split of the quad mesh that provides the full four degrees of freedom despite the projection. The splines complement and are naturally compatible with bi-cubic PHT splines (Deng et al., 2008; Li et al., 2010; Kang et al., 2015) for localized refinement. Fig. 1c illustrates a basis function near a PHT-refined $n = 5$ neighborhood.

1.1. Related literature

In the 1990s, a number of C^1 surface constructions were based on singularities at the vertices (Peters, 1991; Warren, 1992; Pfluger and Neamtu, 1993; Neamtu and Pfluger, 1994; Reif, 1995a, 1997) including constructions of curvature continuous surfaces (Bohl and Reif, 1997; Reif, 1995b). A major contribution of Reif's singular construction (Reif, 1997) was a proof showing that the corner singularity is removable by a local change of variables; and that the resulting surface is tangent continuous at and near the central irregular point where more of fewer than four tensor-product patches come together. More recently, in the context of iso-geometry, Takacs and Jüttler (2012) analyzed singular spline constructions, but did not realize the connection to the earlier surface constructions. They observed that specific linear combinations of singular splines can be sufficiently regular for iso-geometric analysis and closed with the prediction that "main targets for further analysis are approximation properties on singular domains". The monograph (Peters and Reif, 2008) characterizes subdivision surfaces as smooth spline surfaces with singularities at the irregular points and establishes the differential-geometric properties of subdivision surfaces at the singularities. Subdivision functions have repeatedly been used as finite elements (Cirak et al., 2000, 2002; Barendrecht, 2013; Nguyen et al., 2014). The linear independence of Loop and Catmull–Clark subdivision splines, except for the cube mesh, was proven in Peters and Wu (2006).

Reif's construction is based on bicubic splines with double knots. These functions have been generalized for local refinement using T-corners where coarse and fine splines meet. The local refinability of these PHT splines (Deng et al., 2008; Li et al., 2010; Kang et al., 2015) nicely complements the ability we focus on: to create multi-sided blends.

Overview. Section 2 collects the notation and setup for constructing the splines near irregularities. Section 3 derives the splines. Section 4 discusses their properties: C^1 smoothness, refinability and linear independence of the functions associated with the four degrees of freedom of each quadrilateral. Section 5 discusses two uses of the splines.

2. Definitions and setup

We consider a network of quadrilateral facets or *quads*. The nodes where four quads meet are called regular, else *irregular nodes*. An irregular node must not be a direct neighbor of an irregular node, but may be a diagonal neighbor within the same quad. If the assumption fails, one Catmull–Clark-refinement step can enforce the requirement.

Except where the construction switches to the PHT construction to accommodate local refinement, every quad ℓ is associated with four basis functions, hence four degrees of freedom, $c^\ell_{ij} \in \mathbb{R}^d, i, j \in \{1, 2\}$. Surface and analysis space will consist of linear combinations $\sum c_{ij} f_{ij}$ with control points c_{ij}. We obtain basis functions f^k_{ij}, for example, by setting $c^\ell_{ij} = 1$ and all other coefficients to zero and then applying the Algorithm of Section 3. It is convenient to define the basis function f^k_{ij} piecewise by tensor-product polynomials b of bi-degree 3 in Bernstein–Bézier (BB) form

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