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Technical Section

An approach to achieving optimized complex sheet inflation under constraints ☆

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ABSTRACT

Sheet inflation is an enhanced and more general version of the classic pillowing procedure [1] used to modify hexahedral meshes. The flexibility of sheet inflation makes it a valuable tool for hex mesh generation, modification and topology optimization. However, it is still difficult to generate self-intersecting sheet within a local region while assuring the mesh quality. This paper proposes an approach to achieving optimized complex sheet inflation under various constraints. The approach can generate complex sheets that intersect themselves more than once and maximize the quality of the resultant mesh. We successfully apply this approach to mesh matching and mesh boundary optimizing.

1. Introduction

In finite element analysis, hexahedral meshes are usually preferred to tetrahedral meshes due to higher accuracy, faster convergence and lighter storage [2,3]. Therefore, many researchers have been committed to the research of hex meshing for decades. Although there has been tremendous progress in this area, perfect solutions are still eluding for hex mesh generation, modification and topological optimization. The main reason for the difficulties is that local modification inevitably influence the whole hex mesh due to the inherent global connectivity of hex mesh [4–7]. Fig. 1 shows an example that even though we want to make small local modification as adding one new quad to the boundary of the hex mesh (Fig. 1(b)), a circle of quads on the boundary (Fig. 1(c)) as well as a set of hex have to be generated in order to keep the hex mesh valid (Fig. 1(d)). The dual structures, which contribute to the global connectivity of hex meshes, are called sheets. The set of hex in Fig. 1(d) is actually a sheet.

More specifically, starting from one mesh edge, we recursively get all its topologically parallel mesh edges. All of these mesh edges along with the adjacent hex define a sheet. In addition to its primal representation as a composition of vertices, edges, faces and hexahedra, a hex mesh can also be seen as a set of intersecting sheets, known as Spatial Twisted Continuum [4].

Therefore, as a set of operations that directly and effectively deal with the sheets, sheet operations attract more and more attention in recent years. The most common sheet operations include pillowing [1], dicing [8], column collapse [9], sheet extraction [9,10] and sheet inflation [9,11,12].

Sheet inflation takes a continuous quad set as input and generates a new sheet by inflating the quad set. The process of the sheet inflation is illustrated in Fig. 2. This continuous quad set (Fig. 2(b)) is called the quad set for sheet inflation (or quad set if not ambiguous in the context) which is provided by other procedures or specified directly by the user. Sheet inflation duplicates the nodes, edges, and quads on the quadset (Fig. 2(b)) and then forms new hexahedra by connecting the nodes in the quad set to their corresponding duplicates (Fig. 2(c) and Fig. 2(d)). The process of the sheet inflation shows that the quad set is critical for sheet inflation because it determines the position, shape and topology of the sheet to be generated by the sheet inflation. And compared with pillowing, sheet inflation is more flexible and versatile, having the potential ability to create any kind of sheets, provided that a suitable quad set can be determined. Therefore sheet inflation can be used to support various hex mesh modification.

Sheet inflation is a flexible and versatile sheet operation that can create various and complex sheets, and thus it can be utilized to effectively support many hex mesh modification scenarios. One common scenario is to use sheet inflation to locally change the mesh topology, especially the boundary topology of a hex mesh. In this scenario, new sheets usually need to be created to satisfy certain constraints like at the specified position within a delimited region. As the quad set plays a key role for sheet inflation, the

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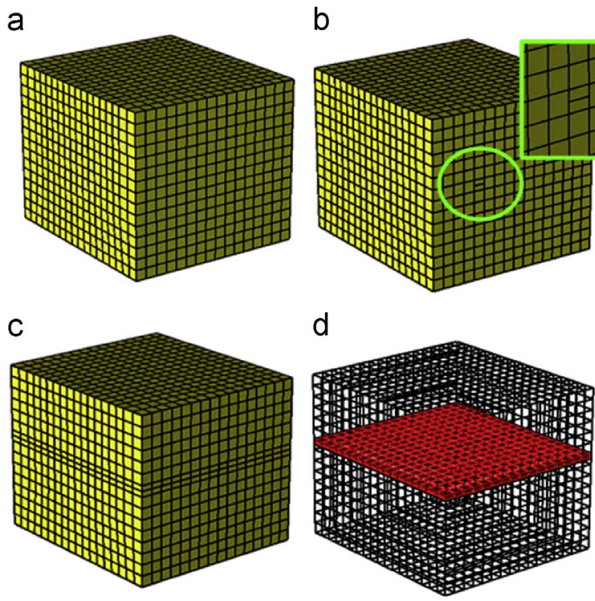

Fig. 1. An example of local modification resulting in global change: (a) the original hex mesh; (b) local modification by adding a quad on the mesh boundary; (c) a circle of quads on the mesh boundary have to be added; (d) a set of hexahedra have to be added.

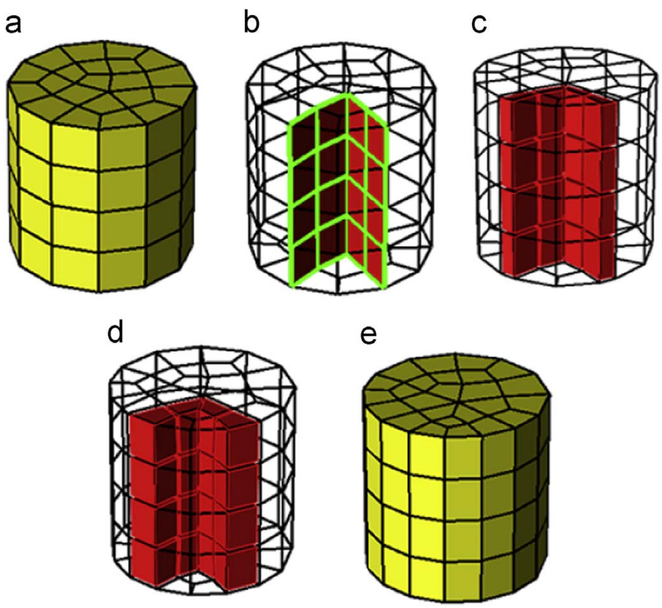

Fig. 2. The procedures of sheet inflation: (a) the hex mesh before sheet inflation; (b) the quad set for sheet inflation; (c) the new sheet is created by inflating the quad set; (d) the new sheet is created; (e) the hex mesh after sheet inflation.

main difficulty to achieve this is how to construct a qualified quad set under the given constraints. In practice, these constraints are normally specified by a set of boundary edges and a set of hexahedra. The former determines the positions where the new sheet should appear on the mesh boundary, and the latter delimits the region where the new sheet belongs to.

Fig. 3 shows an example where such constrained sheet inflation is required. In this example, the hex mesh (Fig. 3(a)) contains one boundary node whose valence is 6 and it is undesirable. As shown in Fig. 3(b), this high valence can be reduced by splitting the node into two nodes with lower valences, and a constrained sheet inflation can be used to achieve this. Specifically, two mesh edges adjacent to the high-valence node are selected as the position

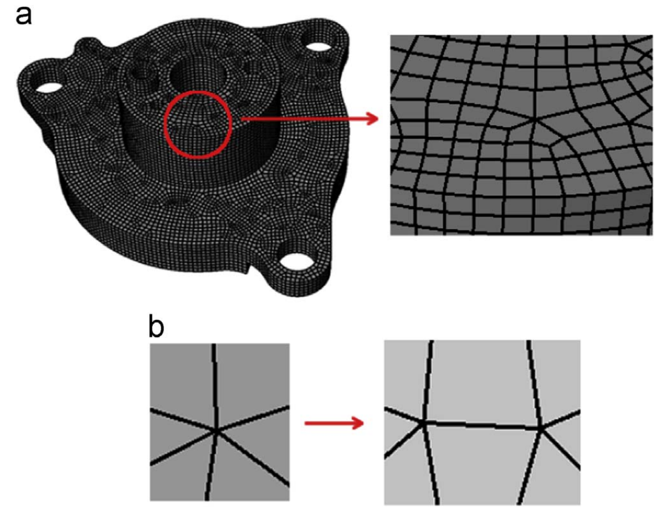

Fig. 3. An example where boundary modification is required: (a) the hex mesh with a high-valence node; (b) the high valence is reduced by splitting the node into two nodes.

constraint of the new sheet (Fig. 4(a)) first, then a quad set is constructed (Fig. 4(b)), finally the new sheet is inflated based on the quad set (Fig. 4(c)). The result is shown in Fig. 4(e).

Without region constraints being specified, the quad set may spread uncontrollably through the hex mesh, e.g. the quad set in Fig. 4(b). Inflating quad sets like this will impact almost the whole mesh. To localize the sheet inflation, region constraints need specifying by a set of hexahedra (yellow in Fig. 5(a)). The quad set is then constructed within that delimited region (Fig. 5(b)). Compared to Fig. 4(d), the localized sheet inflation will impact only a limited part of the hex mesh as shown in Fig. 5(d).

In recognizing the importance of sheet inflation, many researchers have been investigating algorithms for decades and have achieved great progress. Mitchell et al. proposed the Pillowing algorithm to deal with the doublet problem [1]. Its simplicity and effectiveness makes Pillowing prevalent in mesh modification, especially in mesh quality improvement. Although it can be adapted to satisfy simple constraints, Pillowing lacks the ability to generate complex sheets such as self-intersecting sheets.

Suzuki et al. introduced a method to construct interior sheet surfaces when the boundary dual cycles are given [13]. They used this method to first construct dual structures and then deduce the primal hex meshes from these dual structures. While in the paper they illustrated the topological structures of sheet surfaces in the dual space, particularly self-intersecting sheet surfaces, they did not mention how to determine the quad set and generate the corresponding sheets on the primal hex mesh.

Staten et al. proposed the General Sheet Inflation method which explained in detail how to do sheet inflation on hex meshes when boundary mesh edges are specified [9]. This method can create normal, self-touching and self-intersecting sheets. However, Staten provides no details on how to construct quad sets. While his theory supports self-intersecting and self-touching sheets, it is not clear whether his algorithm can generate the required quad sets.

Sheet inflation is an enhanced and more general version of the classic pillowing. Pillowing actually can be seen as a special form of sheet inflation whose quad set is determined by the shared quad set between the shrinking set and the rest of the mesh.

Chen et al. proposed a new approach to inflate sheets when conducting mesh matching on complex interfaces [14]. This method first constructs the boundary loop of the quad set, and then determines the interior quad set. Although it can locally generate self-intersecting sheets under constraints, it allows the