Graphical Models 84 (2016) 15–27

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

Fast computation of optimal polygonal approximations of digital planar closed curves

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ARTICLE INFO

Article history:
Received 27 July 2015
Revised 10 January 2016
Accepted 22 January 2016
Available online 26 February 2016

Keywords:
Digital planar curves
Polygonal approximation
Integral Square Error
Mixed Integer Programming
Discrete optimization
min-# polygonal approximation problem

ABSTRACT

We face the problem of obtaining the optimal polygonal approximation of a digital planar curve. Given an ordered set of points on the Euclidean plane, an efficient method to obtain a polygonal approximation with the minimum number of segments, such that, the distortion error does not excess a threshold, is proposed. We present a novel algorithm to determine the optimal solution for the min-# polygonal approximation problem using the sum of square deviations criterion on closed curves.

Our proposal, which is based on Mixed Integer Programming, has been tested using a set of contours of real images, obtaining significant differences in the computation time needed in comparison to the state-of-the-art methods.

1. Introduction

Since Attneave [1] pointed out that the information is concentrated at dominant points, their detection has become an important research area in computer vision. Dominant points are those that can describe the curve for visual perception and recognition. In the literature two major categories can be found: corner detection methods [2–4] and polygonal approximation methods [5–7].

In computer vision polygonal approximation of digital planar curves is an important task for a variety of applications like simplification on vectorization algorithms [8], image analysis [9], shape analysis [10], object recognition [11], Geographical Information Systems [12], and digital cartography [13].

The main idea behind polygonal approximations of digital planar curves is to provide a compact representation of the original shape with reduced memory requirements, preserving the important shape information.

The optimization problem of polygonal approximations has been formulated into two separated ways, depending on the objective function that we want to minimize:

- min-#: Minimize the number of line segments M that forms a polygonal approximation, such that, the distortion error does not excess a threshold ε. Moreover the optimal solution should have the lowest distortion associated among all the solutions with the same number of line segments.
- min-ε: Given a number of line segments M, minimize the distortion error associated to the polygonal approximation.

In the literature, several alternatives to solve the min-# and min-ε problem can be found, using heuristic, metaheuristic and optimal approaches. The selection of the distortion measure used in the algorithm is task dependent: the use of the L_∞-norm is used to assure that the maximum deviation does not exceed the threshold provided by the user, whereas, the use of the L_2-norm provides a

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http://dx.doi.org/10.1016/j.gmod.2016.01.004
1524-0703/

polygonal approximation whose distortion (Integral Square Error) is lower than the provided threshold.

For instance, to solve the min-ε problem using the L_2-norm several metaheuristics have been applied: genetic algorithms [14], ant colony search algorithms [15], integer particle swarm optimization algorithms [7], etc.

The main problem of using the metaheuristic approaches is the computational cost. There are several proposals in the literature that use some heuristic with a low computational cost: split based methods [16], merge based methods [17,18], merge-split based methods [5], etc.

This problem is solved optimally by using dynamic programming approach [19] and graph approach [6]. These algorithms obtain optimal solutions, however a high computational burden is required to obtain them. A more recent and faster method based on Mixed Integer Programming was proposed in [20].

In practice, the reduction of the description of a shape with a maximum tolerance error, that is, the min-# problem, is a more common task. Therefore, a great variety of algorithms have been proposed to solve this problem. For example, the min-# problem using the L_2-norm has been solved by using metaheuristic solutions: genetic algorithms [21], ant colony optimization [15], particle swarm optimization [22,23], tabu search [24], etc. More rapid algorithms are based on other heuristics like a split approach [13], merge approach [25], graph approach [26] etc.

The min-# can also be solved optimally by using a modified version of the dynamic programming approach proposed by Perez and Vidal [19] and a graph approach by Salotti [27]. The original algorithm, by Perez and Vidal, was used to obtain the polygonal approximation with a fixed number of segments M which has the minimum distortion error associated (min-ε problem). This modified version of the algorithm was proposed in [27] and is shown in Algorithm 1. The idea is to increase the number of segments needed to reach the last point of the curve until the error associated to the polygonal approximation is lower than a threshold ε.

The main drawback of using optimal algorithms is the computational cost required to achieve the solution. Some improvements have been made to reduce this computational burden. For instance, Horng and Li [28] proposed an heuristic method to determine the initial point for the method based on Dynamic Programming used to solve the min-ε. This method uses two iterations of the Dynamic Programming method to obtain a polygonal approximation. Kolesnikov and Fränti [29] introduced a method to obtain polygonal approximations based on a cyclically extended closed curve of double size. The method selects the best starting point by searching on the extended search space for the extended curve. Both methods obtain good solutions using different heuristic approaches, however, neither of these methods can assure the optimality of the solution.

However, optimal polygonal approximations are very important because they are commonly used to assess the quality of the suboptimal polygonal approximations obtained by suboptimal methods.

The main contributions of this paper are: (i) a novel Mixed Integer Programming (MIP) model to solve the min-# polygonal approximation problem using a different approach to the state-of-the-art optimal algorithms, (ii) our proposal reduces significantly the computation time to obtain the optimal solution; (iii) the proposed MIP model is smaller than previous proposals.

Algorithm 1: Modified version of the algorithm by Perez and Vidal [19].

```
Data: c (Digital planar curve), ε (maximum error)
Result: The optimal polygonal approximation
var NP // Number of points;
var NS // Maximum number of segments;
var g // used to memorize the minimum global error
to reach any point of the contour using any number of
segments;
var Points // Points of the digital planar curve;
var Father // Array that contains the ending point of
the previous segment;
g[1, 0] ← 0;
for n ← 2 to NP do
    g[n, 0] ← maxValue;
end
m ← 0;
repeat
    m ← m + 1;
    for n ← 2 to NP do
        // Search the minimum error to reach point n
        with m segments;
        g[n, m] ← min(g[i, m − 1] + ISE(i, n));
        // Store the index i_min of the point with the
        minimum error;
        Father[n, m] ← i_min;
    end
until g[m, n] < ε;
```

Section 2 summarizes the most important measures to evaluate the quality of polygonal approximations that appear in the literature. In Section 3, we formulate the problem and present the MIP model that solves this problem. Section 4 defines the experiments carried out and the results obtained. In Section 5, we discuss the results obtained and the most important aspects of the proposed method. Finally, Section 6 presents the main conclusions of this paper.

2. Measures to assess the quality of polygonal approximations

The quality of a polygonal approximation is quantified by the amount of data reduction obtained and the closeness of the approximation to the original curve. Several authors have faced this problem using different approaches.

Sarkar [30] proposed a method to evaluate the quality of the polygonal approximation, based on the distortion associated to the solution (Integral Square Error) and the compression ratio obtained by the solution. This measure, called Figure of Merit (FOM), is defined as

$$\mathrm{FOM} = \frac{\mathrm{CR}}{\mathrm{ISE}} \tag{1}$$