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Multiresolution on spherical curves

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ABSTRACT

In this paper, we present an approximating multiresolution framework of arbitrary degree for curves on the surface of a sphere. Multiresolution by subdivision and reverse subdivision allows one to decrease and restore the resolution of a curve, and is typically defined by affine combinations of points in Euclidean space. While translating such combinations to spherical space is possible, ensuring perfect reconstruction of the curve remains challenging. Hence, current spherical multiresolution schemes tend to be interpolating or midpoint-interpolating, as achieving perfect reconstruction in these cases is more straightforward. We use a simple geometric construction for a non-interpolating and non-midpoint-interpolating multiresolution scheme on the sphere, which is made up of easily generalized components and based on a modified Lane–Riesenfeld algorithm.

1. Introduction

The question of how to decrease the resolution of a curve and restore it to its original state is a well-studied subject in computer graphics, and falls under the purview of *multiresolution frameworks*. Applications include level-of-detail control, compression, and multiscale editing for curves. Such frameworks can be created using a combination of subdivision and reverse subdivision [1].

In Euclidean space, subdivision schemes are linear transformations that increase the resolution of a curve or surface, while reverse subdivision schemes are linear transformations that decrease the resolution. Many subdivision schemes are based on B-Spline basis functions, and converge to B-Spline curves or surfaces at the limit. Chaikin's corner-cutting scheme for curves [2] as well as the Catmull–Clark [3] scheme for surfaces are some well-known examples of B-Spline subdivision schemes for which reverse methods have been proposed. Both forward and reverse subdivision are often understood and implemented using affine combinations of points, specified by simple linear filters.

When combined into a multiresolution framework, a given vector of m fine points $f = [f_0 \dots f_{m-1}]^T$ can be decomposed to a vector of $n < m$ coarse points $c = [c_0 \dots c_{n-1}]^T$ and associated detail vectors (or wavelet coefficients) $d = [d_0 \dots d_{m-n-1}]^T$ [4,5], then reconstructed using c and d. A notable property of such a framework is that the total number of coarse points and details is equal to the original number of points before decomposition. As a result, no additional information is needed to fully retrieve the high resolution data. Furthermore, these operations are both fast and efficient.

While well understood in 2D or 3D Euclidean space, achieving multiresolution via subdivision and reverse subdivision in other spaces is a challenging but fascinating topic of study. The sphere, for instance, is an elegant and important geometric domain, and of particular interest as an approximation of the shape of the Earth [6]. However, its surface forms a two-dimensional non-Euclidean space in which many traditional geometric intuitions do not apply. Curves in spherical space — analogous to curves in Euclidean space — are called spherical curves and are formed by an ordered set of points f_i on the sphere connected by geodesic lines (great circle arcs).

Our work focuses on decreasing and increasing/restoring the resolution of spherical curves (i.e. spherical multiresolution) based on B-Spline subdivision and reverse subdivision, with an intended application in vector data representation on the spherical surface of a Digital Earth [7,8]. Geospatial vector data are often very large (consisting of thousands of points) and can benefit from multiscale representations due to their support for compression, progressive transmission over networks, level-of-detail control in visualization, and fast estimates for queries.

In general, the fundamental challenge in spherical multiresolution lies in translating affine combinations of points to spherical space in a manner that ensures the scheme is loss-less (i.e. perfect reconstruction of the original fine data f is achieved).

A straightforward solution is to project the points of the spherical curve to a Euclidean domain (e.g. using a spherical projection from the field of cartography), apply affine combinations in that domain, and project back to the sphere. Potential mappings

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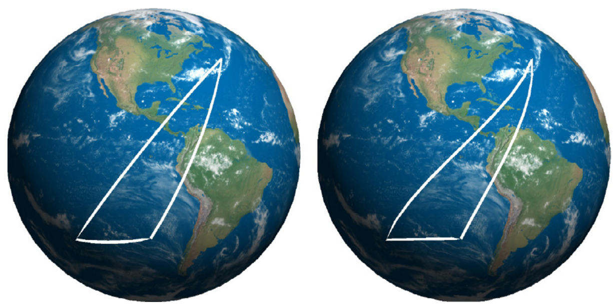

Fig. 1. A spherical curve defined by three points is shown (on left). After mapping the points to latitude/longitude coordinates, drawing Euclidean lines between the resulting points, and mapping those lines back to the sphere, significant mapping distortions are revealed (on right).

include latitude/longitude or spherical coordinate conversion, which is a standard projection; Snyder projection [9], which is an equal area projection often encountered in Digital Earth frameworks; and the exponential map [10], which maps points to a local tangent plane. Unfortunately, as the spherical and Euclidean space are not isometric, this approach often introduces distortions into the resulting curves (see Fig. 1).

A second approach is to generalize the affine combination $p = a_0 q_0 + a_1 q_1 + \cdots + a_{n-1} q_{n-1}$ (p, $q_i \in \mathbf{R}^3$, $a_i \in \mathbf{R}$) to spherical space as the (local) solution to

$$\min_p \left\| \sum_{i=0}^{n-1} a_i \cdot \exp_p(q_i) \right\|,$$

as in [11], where $\exp_p(q_i)$ is the exponential map operator that maps q_i to a vector in the tangent space of p. However, due to the nature of non-Euclidean space, generalizing these combinations in this manner does not in general result in a scheme with perfect reconstruction. Hence, the work of [11] focuses on interpolating and midpoint-interpolating multiresolution schemes, for which perfect reconstruction can be guaranteed. As a non-interpolating and non-midpoint-interpolating (i.e. approximating) scheme, B-Spline multiresolution is non-trivial to translate to spherical space.

A third approach, as seen in [12,13] and the one adopted in this paper, is to split the affine combinations into series of two-point interpolations. Such two-point interpolations are atomic operations in spherical space that are analogous to the simplest atomic operations used to create curves in Euclidean space, and can be computed efficiently using spherical linear interpolation (SLERP), defined by [14]

$$\mathrm{SLERP}(p, q, u) = \frac{\sin[(1-u)\theta]}{\sin(\theta)} p + \frac{\sin(u\theta)}{\sin(\theta)} q$$

(where θ is the angle between p and q). Unlike Euclidean space, in which any reformulation of an affine combination into two-point interpolations will have the same result, in spherical space different reformulations of the affine combination give different results. Hence, it is again difficult to ensure perfect reconstruction in this case.

We present in this paper a construction of a loss-less approximating multiresolution scheme in spherical space (inspired by Euclidean B-Spline multiresolution) made up of sequences of two-point interpolations (i.e. SLERP operations). This holds for all constituent operations of the multiresolution: subdivision, reverse subdivision, detail computation (i.e. decomposition), and detail restoration (i.e. reconstruction). The construction is inspired by the Lane–Riesenfeld subdivision algorithm for B-Spline subdivision of arbitrary degree (or smoothness) in Euclidean space [15], which uses two atomic operations: point duplication and midpoint finding. Although easily generalized to the sphere, the algorithm does not have a corresponding reverse subdivision or multiresolution algorithm due to the non-invertibility of the midpoint finding operation.

Our construction, which can reproduce at least some of the B-Spline subdivisions returned by the Lane–Riesenfeld algorithm, replaces pairs of midpoint-finding operations with discrete smoothing operators that have local inverses in Euclidean and spherical space. Detail vectors d_i are generalized to detail *rotations* in spherical space, and are easy to compute and restore during reconstruction.

Furthermore, our multiresolution scheme includes reverse subdivision, detail computation, and detail restoration constructions based on atomic operations; to our knowledge the first of their kind. We expect translations of this scheme to more general manifolds are possible as well, provided an operation analogous to SLERP is defined on the manifold.

The paper is organized as follows. In Section 2, we describe previous works that are related to this problem. A generalization of the Lane–Riesenfeld algorithm to spherical space from [13] is described in Section 3, followed by a generalization to spherical space of the modified Lane–Riesenfeld algorithm with invertible averaging step from [16] in Section 4. In Sections 5, 6, and 7, we present our spherical multiresolution scheme, with some comments on analysis in Section 8. Results and comparisons follow in Section 9.

2. Related work

Curves that lie on surfaces (including spheres) have been the subject of much research [7,8,17]. Spherical curves are especially important, as the sphere is an important shape in Geomatics and GIS and serves as an important intermediate shape for applications such as parametrization and illumination [18,19]. Spherical curves are particularly of interest within the Digital Earth framework [6,20–22], which represents the Earth as a curved surface rather than as a flattened map.

Multiresolution for curves and surfaces is also a well-studied subject [23–25]. One means of establishing a multiresolution framework is to combine subdivision and reverse subdivision, in which the former produces a more detailed object while the latter reduces the resolution [4,5,26]. The convergence and smoothness of the limit curve of a subdivision scheme can be analysed using the techniques in [27] for Euclidean space and [12,28] for manifold surfaces. In a multiresolution framework based on subdivision and reverse subdivision, no details are lost and all information needed to reconstruct the curve occupies no more memory than the original model.

These methods are usually understood and implemented in terms of affine combinations/weighted averages. The taking of an affine combination in Euclidean space is a fundamental operation and very useful for efficient geometric processing. As a result, redefining weighted averages within the manifold, spherical, and Riemennian spaces have been studied in several previous works [29,30].

Affine combinations on the sphere have been approached via iterative optimization [8]. However, since the exact results of the weighted averages in this method are not known *a priori* (due to iterative solving of the optimization), we cannot develop a loss-less multiresolution scheme based on this method in the approximating case.

The coefficients of an affine combination may be used as barycentric coordinates to describe a point with respect to a set of polygon vertices. The spherical barycentric coordinates of a point p inside a spherical triangle may be calculated using the method described in [17], or for a point p inside a spherical polygon using the work of [31]. In [17], the resulting barycentric coordinates may be used to represent p as a linear combination of the vertices of