Graphical Models 78 (2015) 1–9

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

Simple and branched skins of systems of circles and convex shapes ☆

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ARTICLE INFO

Article history:
Received 14 July 2014
Received in revised form 21 October 2014
Accepted 2 December 2014
Available online 10 December 2014

Keywords:
Circle
Convex shape
Skinning
Interpolation
Spline

ABSTRACT

Recently, there has been considerable interest in skinning circles and spheres. In this paper we present a simple algorithm for skinning circles in the plane. Our novel approach allows the skin to touch a particular circle not only at a point, but also along a whole circular arc. This results in naturally looking skins. Due to the simplicity of our algorithm, it can be generalised to branched skins, to skinning simple convex shapes in the plane, and to sphere skinning in 3D. The functionality of the designed algorithm is presented and discussed on several examples.

1. Introduction

This paper is devoted to an interesting geometric operation, the operation of skinning. Skinning is a construction of a G^k (C^k) continuous interpolation curve/surface of an ordered sequence of planar or spatial shapes (most often of the same type such as circles and spheres). This operation can be viewed as a particular analogy to the well-known interpolation of point data sets, which is one of the most crucial techniques in Computer Aided Geometric Design, cf. [1] and the references therein. Due to its technical importance, skinning has attracted the geometric modelling community in recent years and one can find several papers on this topic, see e.g. [2–4]. This type of interpolatory skinning is not to be confused with skinning in graphics applications such as [5].

Circles and spheres belong among the most used input shapes for skinning, as these objects and their skins play an important role in various applications, e.g. computational chemistry, molecular biology, computer animation, and modelling of tubular surfaces, cf. [6,7]. In a discrete sense, skinning can be regarded as a part of the problem of computing envelopes of families of circles/spheres using the cyclographic mapping [8,9]. Skinning is also closely related to representing shapes with the help of the associated medial axis/surface transforms [10–15] and the theory of canal and pipe surfaces [16–18].

The problem of skinning is defined and solved ambiguously in the literature: for a given configuration of input shapes, there exist infinitely many potential solutions. For instance, several techniques exist that skin a molecular model, producing a C^1 continuous surface of high quality. These methods take a cloud of circles/spheres as input and their connectivity is decided automatically based on Voronoi and Delaunay complexes, see [19–21]. In contrast, our input is a *connected* system of input shapes and the resulting skin respects the input connectivity. Some authors impose additional conditions on skin constructions, e.g. minimal surface area or minimal mean curvature [3]. Others prefer simplicity of skinning algorithms; our approach belongs to this category. For instance, an elegant

☆ This paper has been recommended for acceptance by Peter Lindstrom.

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http://dx.doi.org/10.1016/j.gmod.2014.12.001
1524-0703/

and purely geometric method based on constructing Apollonius circles [22] has been used recently for finding skin touching points on input circles/spheres [4]. Furthermore, different algorithms use different admissible input data sets [23].

All the techniques mentioned above assume that the skin touches each of the input planar (spatial) shapes at two points (along a curve). Our main motivation is to design a simple and efficient algorithm which allows skins to consist also of parts of the boundaries of the input shapes. In our opinion, this introduces an intuitive physical interpretation of skinning which simulates enclosing a sequence of planar shapes by a rubber band (i.e., a generalisation of a spline in the original sense). On the other hand, the modelling adaptivity of our method (i.e., the use of weights) enables the user to shrink the arcs to points (or spherical parts to curves) and thus to obtain a skin in the usual sense. In addition, the simplicity of our algorithm enables us to generalise it directly to the construction of branched skins, to skinning simple convex shapes in the plane, and to sphere skinning in 3D.

The rest of the paper is organised as follows. After introducing our approach to skinning and basic definitions in Section 2, we introduce our skinning algorithm in Section 3. Examples of the basic algorithm are presented in Section 4, examples of its generalisations are shown in Section 5. In Section 6, we discuss the novelty of our approach, and we conclude the paper in Section 7.

2. Preliminaries

We recall some elementary notions and address the circle skinning problem. In the following sections this problem will be generalised to other convex shapes in the plane and to spheres in space.

We start by defining our notation. We denote c a positively oriented circle in the plane parametrised such that when travelling along the circle one always has its interior to the left, and d the corresponding disk whose boundary is c. Analogously, in what follows, positive orientation is always considered in the anticlockwise sense. Furthermore, when speaking about tangents and tangent vectors of circles, we assume that their orientation follows the anticlockwise orientation of the circles.

Following [4], we call a sequence $C = \{c_1, c_2, \dots, c_n\}$, $n \in \mathbb{N}$, of (oriented) circles *admissible* (see Fig. 1) if the corresponding sequence of disks $D = \{d_1, d_2, \dots, d_n\}$ satisfies the conditions:

$$\begin{aligned} &\text{(i)} \quad d_i \not\subset \bigcup_{j=1, j\neq i}^{n} d_j, \quad i \in \{1,2,\dots,n\}, \\ &\text{(ii)} \quad d_{i-1} \cap d_{i+1} \neq \emptyset \Rightarrow d_{i-1} \cap d_{i+1} \subset d_i. \end{aligned} \tag{1}$$

The relation $X \prec_c Y$ between two points X and Y on c reflects the direction of the parameter along c (considered wrapped around when necessary). The symbol $\widehat{XY}$ denotes the oriented arc on c delimited by the points X, Y such that $X \prec_c Y$. The *midpoint* Z of the arc $\widehat{XY} \subset c$ is defined by the property $Z \in \widehat{XY}$, $\|X - Z\| = \|Z - Y\|$, and denoted $Z = X \overset{\bullet}{\frown}_c Y$.

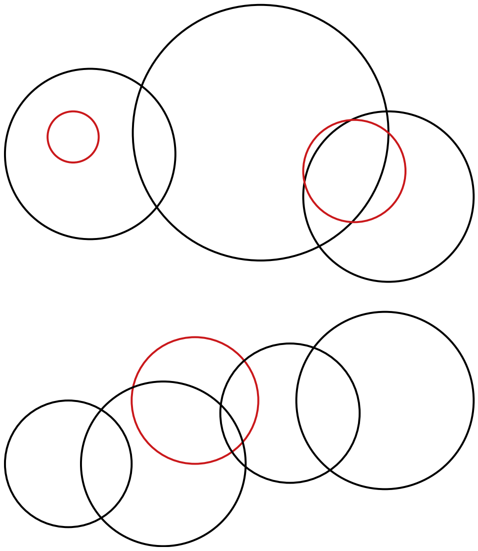

Fig. 1. Non-admissible sequences of circles. Top: The circles in red violate condition (i). Bottom: The circle in red violates condition (ii) in (1). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

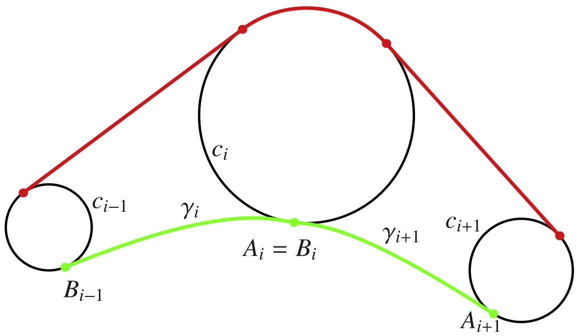

Fig. 2. Three consecutive circles c_{i-1}, c_i, c_{i+1} of C, the contact points on them, and the interpolating splines between them.

We now define the operation of *skinning*. Somewhat unconventionally, we first define only one branch of the skin of C, the one on the right; see the green[1] (bottom) curve in Fig. 2. This branch is denoted by $\mathcal{B}(C)$ and called the *skin branch* of C. The reason for this approach becomes apparent in Section 5.2, where we generalise the linear sequence C of input circles to a branched system of circles.

For each circle $c_i \in C$, we define its right *contact arc* $a_i = \widehat{A_iB_i}$, $A_i \prec_{c_i} B_i$, as an arc on c_i along which the skin branch is to touch it; see Fig. 2. These arcs are allowed to degenerate into points (i.e., $A_i = B_i$ for some or all i). The computation of these points is discussed in detail in Section 3.1. Our approach generalises previous definitions of skins [2,4].

A G^1 skin branch $\mathcal{B}(C)$ of an admissible sequence C of circles is a G^1 spline such that

$$\mathcal{B}(C) = B_1 \gamma_2 a_2 \dots a_{n-1} \gamma_n A_n, \tag{2}$$

[1] For interpretation of colour in Fig. 2, the reader is referred to the web version of this article.