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Reducing numerical dissipation in smoke simulation ☆

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ABSTRACT

Numerical dissipation acts as artificial viscosity to make smoke viscous. Reducing numerical dissipation is able to recover visual details smeared out by the numerical dissipation. Great efforts have been devoted to suppress the numerical dissipation in smoke simulation in the past few years. In this paper we investigate methods of combating the numerical dissipation. We describe visual consequences of the numerical dissipation and explore sources that introduce the numerical dissipation into course of smoke simulation. Methods are investigated from various aspects including grid variation, high-order advection, subgrid compensation, invariant conservation, and particle-based improvement, followed by discussion and comparison in terms of visual quality, computational overhead, ease of implementation, adaptivity, and scalability, which leads to their different applicability to various application scenarios.

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1. Introduction

Smoke is desirable in visual effect and video game industries. It is also one of challenging problems in computer graphics due to its complexity and turbulence. To obtain realistic smoke and gaseous phenomena, physically based methods with Navier–Stoke Equations (NSEs) have been explored to model underlying fluid dynamics. Although numerically integrating NSEs have been studied in computational fluid dynamics (CFD), computer graphics researches focus on simplified discretization and numerical schemes when visual quality matters most. Simplifications make physically based methods possible for smoke simulation but introduce the numerical dissipation. The numerical dissipation increases fluid viscosity to make it appear more viscous than intended. It degrades the visual appearance by smearing out fine details and damping down the motion quickly. The numerical dissipation has been recognized to have substantial visual consequences to the smoke simulation.

Many sources introduce numerical dissipation to the course of the smoke simulation. Coarse spatiotemporal discretization produces numerical truncation errors, which is proven to have a form of viscosity [1]. As fluid quantities are only defined on discrete locations such as grid points and particles, interpolation schemes are required to calculate values at undefined positions, which is equivalent to smoothing operations that produce the numerical dissipation. The semi-Lagrangian method [2] is widely used for the smoke simulation attributed to its unconditional stability and ease of implementation, but it generates a large amount of the numerical dissipation in backward tracing and advection subroutines. Many advanced methods are constructed based on the semi-Lagrangian method to guarantee the unconditional stability. However, they also inherit the disadvantage of massive numerical dissipation.

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Massive effort has been devoted to combat the numerical dissipation from different aspects. Some methods are developed to eliminate sources of the numerical dissipation. For instance, it is straightforward to reduce the numerical dissipation by increasing spatial resolution and reducing time step. However, both approaches increase computational overhead. Adaptive mesh [3], irregular mesh [4,5], and dynamical mesh [6] are proposed to reduce the numerical dissipation without significantly increasing computation. Rather than directly reducing the numerical dissipation, several methods generate artificial details to compensate for visual loss using vorticity confinement [7,8] and subscale turbulence models [9,10]. Grid-based methods require resampling flow field, which is equivalent to the low-pass filter to smear out high-frequency components. Particle-based methods only carry quantity but do not dissipate quantity, which does not suffer from the numerical dissipation problem. However, particle methods have problems such as particle redistribution. Hybrid particle and grid methods [11,12] are proposed to leverage advantages of particle and grid to reduce numerical dissipation.

In this paper, we investigate the numerical dissipation in smoke simulation in terms of where it comes out, what impact it has, and how to combat it. The rest of paper is organized as follows: we give a brief introduction to the basic smoke simulation in Section 2 and address the sources of the numerical dissipation in Section 3; in Section 4 we investigate and compare methods of combating numerical dissipation from different aspects, following by a conclusion in Section 5.

2. Background

Smoke and other gaseous phenomena are normally simplified to be incompressible and homogenous, which does not decrease the applicability to model basic dynamical mechanisms. The NSEs to model smoke are derived as:

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} + \nu\nabla^2\mathbf{u} - \frac{\nabla p}{\rho} + \mathbf{f} & \text{(a)} \\ \nabla\cdot\mathbf{u} = 0 & \text{(b)} \end{cases} \tag{1}$$

where $\mathbf{u}$ is velocity, p and ρ denote pressure and fluid density respectively. ν is kinematic viscosity to measure how viscous the fluid is and $\mathbf{f}$ represents the resultant external force. The two equations indicate that the fluid should conserve both momentum and mass. The first equation is derived from Newton's second law with left-hand term presenting acceleration and right-hand terms the net force exerted on fluid.

NSEs are too complicated to solve for analytical solution directly. The NSEs usually break down into simple terms including advection, pressure, diffusion, and external force [2]. The simple terms can then be easily solved individually. If we define the terms as operators denoted by $\mathbb{A}$, $\mathbb{P}$, $\mathbb{D}$, and $\mathbb{F}$, the operator $\mathbb{S}$ to solve NSEs can be written as [13]:

$$\mathbb{S} = \mathbb{P}\circ\mathbb{F}\circ\mathbb{D}\circ\mathbb{A} \tag{2}$$

where

$$\begin{cases} \mathbb{A}: \frac{\partial q}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{q} & \text{(a)} \\ \mathbb{D}: \frac{\partial \mathbf{u}}{\partial t} = \nu\nabla^2\mathbf{u} & \text{(b)} \\ \mathbb{F}: \frac{\partial \mathbf{u}}{\partial t} = \mathbf{f} & \text{(c)} \\ \mathbb{P}: \frac{\partial \mathbf{u}}{\partial t} + \frac{\nabla p}{\rho} = 0, \text{ so that } \nabla\cdot\mathbf{u} = 0 & \text{(d)} \end{cases} \tag{3}$$

where q can be velocity, temperature, or any other fluid quantity.

3. Numerical dissipation as artificial viscosity

Numerical solutions are different from exact solution due to numerical truncation errors. The truncation errors include additional high-order terms which influence fluid motion and appearance. We start with the simple one-dimensional advection to analyze the impact on fluid motion:

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} = 0,\ u > 0 \tag{4}$$

If we discretize it using forward Euler for the time derivative and first-order backward difference for the space derivative we can get:

$$\frac{q_i^{n+1} - q_i^n}{\Delta t} + u\frac{q_i^n - q_{i-1}^n}{\Delta x} = 0 \tag{5}$$

We can rearrange it to get

$$q_i^{n+1} = q_i^n - \Delta t\frac{q_i^n - q_{i-1}^n}{\Delta x}u \tag{6}$$

Recalling the Taylor series for q_{i-1}^n gives

$$q_{i-1}^n = q_i^n - \left(\frac{\partial q}{\partial x}\right)_i^n \Delta x + \left(\frac{\partial^2 q}{\partial x^2}\right)_i^n \frac{\Delta x^2}{2} + O(\Delta x^3) \tag{7}$$

Substituting it into above equation and doing the cancelation gives

$$q_i^{n+1} = q_i^n - \Delta x\left(\frac{\partial q}{\partial x}\right)_i^n u + \Delta t\Delta x\left(\frac{\partial^2 q}{\partial x^2}\right)_i^n u + O(\Delta x^2) \tag{8}$$

Deleting the second-order truncation error and rewriting it gets

$$\frac{q_i^{n+1} - q_i^n}{\Delta t} + u\left(\frac{\partial q}{\partial x}\right)_i^n = \Delta x\left(\frac{\partial^2 q}{\partial x^2}\right)_i^n u \tag{9}$$

Which is the forward Euler in time applied to the modified PDE

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} = u\Delta x\frac{\partial^2 q}{\partial x^2} \tag{10}$$

The Laplacian of q in one dimension is $\nabla^2 q = \partial^2 q/\partial x^2$. Defining $\nu' = u\Delta x$ and substituting it into the equation gives

$$\frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} = \nu'\nabla^2 q \tag{11}$$

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