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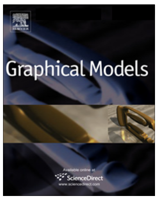

Balanced multiresolution for symmetric/antisymmetric filters [☆],[☆☆]

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ABSTRACT

Given a set of symmetric/antisymmetric filter vectors containing only regular multiresolution filters, the method we present in this article can establish a *balanced multiresolution* scheme for images, allowing their *balanced decomposition* and subsequent perfect reconstruction without the use of any extraordinary boundary filters. We define balanced multiresolution such that it allows balanced decomposition i.e. decomposition of a high-resolution image into a low-resolution image and corresponding *details* of equal size. Such a balanced decomposition makes on-demand reconstruction of *regions of interest* efficient in both computational load and implementation aspects. We find this balanced decomposition and perfect reconstruction based on an appropriate combination of symmetric/antisymmetric extensions near the image and detail boundaries. In our method, exploiting such extensions correlates to performing sample (pixel/voxel) split operations. Our general approach is demonstrated for some commonly used symmetric/antisymmetric multiresolution filters. We also show the application of such a balanced multiresolution scheme in real-time focus+context visualization.

1. Introduction

1.1. Context

Applications that facilitate multiscale 2D and 3D image visualization and exploration (see [17,34,28], for example) benefit from multiresolution schemes that decompose high-resolution images into low-resolution approximations and corresponding *details* (usually, wavelet coefficients). Several subsequent applications of such a decomposition constructs the corresponding *wavelet transform*. This wavelet transform can then be used to derive low-resolution approximations of the entire image, as well as high-resolution approximations of a *region of interest* (ROI), on demand. Reconstructing the high-resolution approximation of a ROI involves locating the corresponding details from a hierarchy of details within the wavelet transform. One such hierarchy of details resulting from only two levels of decomposition of an Earth image (data source: Visible Earth, NASA) is shown in Fig. 1.

For the purpose of demonstration, we created the wavelet transform in Fig. 1 using the *short* filters of quadratic B-spline presented by Samavati et al. [27,28]. In practice, images that require multiscale visualization are larger in size and may require more levels of decomposition. For each level of decomposition in this particular example, the image was first decomposed heightwise and then widthwise.

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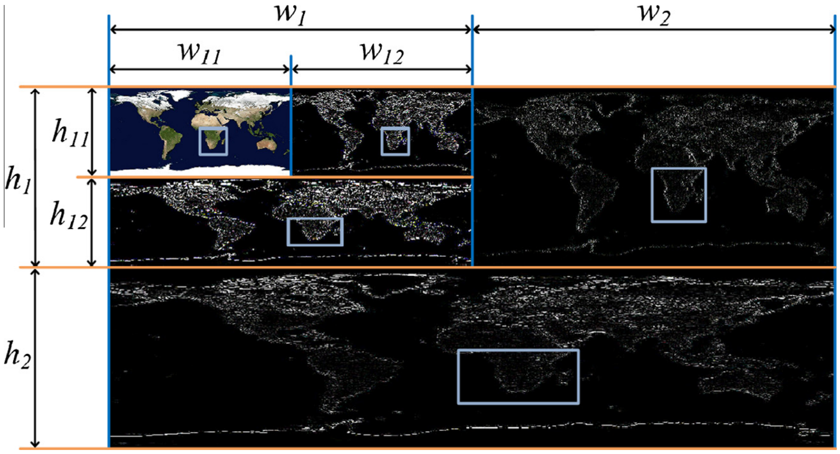

Fig. 1. Hierarchy of details in a wavelet transform resulting from two levels of decomposition of a 1024 × 512 Earth image. The coarse image (at the top left corner) contains a rectangular ROI and the details corresponding to that ROI are enclosed by rectangles within all levels of details.

1.2. Problem

Sequences of samples along each image dimension can be treated as finite-length signals. It is well-known that decomposition and reconstruction of finite-length signals require special treatments at the boundaries [1], which often involves the use of extraordinary boundary filters. The use of extraordinary boundary filters (as opposed to regular filters) for handling image and detail boundaries lead to computationally untidy reconstruction near image boundaries.

From a hierarchy of details, such as the one in Fig. 1, if we need to reconstruct the high-resolution approximation of a ROI located in the low-resolution (coarse) image shown in the top-left rectangle in Fig. 1, we have to locate the corresponding details in some or all of the rectangles that contain details depending on the expected level of resolution. Locating these details will be straightforward if each of the heightwise and widthwise decomposition steps decomposes an image into two halves of equal size – one half corresponding to the coarse image and the other half corresponding to the details. Among B-spline wavelets, only the filters obtained from Haar wavelets provide such a balanced decomposition [12,29]. However, because Haar wavelets and the associated scaling functions are not continuous, it would be beneficial to achieve such a balanced decomposition for the filters obtained from higher order scaling functions and their wavelets.

Existing multiresolution schemes for the local filters of second or higher order scaling functions and their wavelets (see [28,5,8,21], for example) result in unequal numbers of coarse and detail samples after decomposition (i.e. $w_1 \neq w_2, w_{11} \neq w_{12}, h_1 \neq h_2$, and $h_{11} \neq h_{12}$ in Fig. 1). Such inequalities resulting from decomposition make locating the details corresponding to a ROI for reconstruction a cumbersome task (which involves keeping track of level-wise offsets from boundaries), specially when an interactive multilevel visualization hierarchy (see Fig. 13(a), for example) is concerned. Creation of a such an interactive visualization hierarchy requires efficient on-demand access to details.

In contrast, balanced decompositions can construct *balanced wavelet transforms*, such as the one shown in Fig. 2 (data source: Visible Earth, NASA). In Fig. 2, the rectangles containing different levels of details for the entire image are numbered with $(l,1)$ tuples for widthwise and $(l,2)$ tuples for heightwise decompositions, where l represents the level of resolution. Locating the details corresponding to a ROI on demand in a balanced wavelet transform includes a number of simple dyadic operations, which are known to perform significantly faster than non-dyadic operations in both hardware and software implementations. Such efficient access to details is demonstrated by means of an example in Fig. 2. In general, if $c_{x,y}$ is the coarse sample at the top-left corner of a ROI rectangle, then $d^{(l,1)}_{2^{l-1}(w_c+x),2^{l-1}y}$ and $d^{(l,2)}_{2^l x,2^{l-1}(h_c+y)}$ are the detail samples at the top-left corners of the detail rectangles corresponding to the ROI for widthwise and heightwise balanced decompositions, respectively. Here, $w_c \times h_c$ ($\frac{w}{4} \times \frac{h}{4}$ in Fig. 2) is the resolution of the coarse image containing the ROI.

1.3. Proposed approach

In order to address the issues discussed above, in this article, we introduce a technique for devising *balanced multiresolution* schemes for the local filters of second or higher order scaling functions and their wavelets. Our technique uses an appropriate combination of symmetric/antisymmetric extensions near the image and detail boundaries, which correlate to sample split operations. To guarantee a perfect (lossless) reconstruction without the use of any extraordinary boundary filters, our method requires each of the given decomposition and reconstruction filter vectors (kernels) to be either symmetric or antisymmetric about their centers. Many existing sets of local regular multiresolution filters, such as those associated with the B-spline wavelets [28], biorthogonal and