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Using the fully developed concentration profile to determine particle penetration in a laminar flow tube

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ABSTRACT

A previously found analytical expression for the fully developed concentration profile of aerosol particles flowing through a circular tube under laminar conditions has been used to derived a simple analytical expression to determine particle penetration. The resulting expression, $P = 0.846 \exp(-3.659\beta)$ where β is the dimensionless diffusion coefficient of the particle, has been compared satisfactorily with the well-known Gormley–Kennedy equation even for values of β as low as 0.04 in spite that, in principle, the approximate expression for the fully developed concentration profile is only valid for $\beta > 0.2$.

1. Introduction

Knowledge of particle penetration through a tube, i.e. the fraction of particles surviving deposition to the tube wall, is required in many applications of aerosol science and technology. Among the variety of mechanisms by which loss of aerosol particles can occur, the one that so far has attracted most of the research efforts is diffusion. According to this mechanism, small particles undergoing random displacements as a consequence of the ceaseless bombardment of the surrounding gas molecules deviate from their otherwise deterministic trajectories given by the fluid streamlines. Whenever a wandering particle reaches the tube wall it gets attached to it, mainly by Van der Waals forces, and is lost.

The vast majority of the earlier papers published on the subject were devoted to the solution of the convection-diffusion equation for the common case in which particle axial dispersion can be neglected (Davies, 1973; Gormley & Kennedy, 1949; Ingham, 1975; and many others). Experimental studies on particle diffusion in laminar flow tubes have also received great attention in the past, and have served to confirm the prediction of the theoretical approaches. In particular, aerosol penetration through tubes has been exhaustively measured and found to be in excellent agreement with the series solution proposed by Gormley and Kennedy (1949). Later works allowed verification of this series solution for very small particles with diameters down to 2–3 nm and even for ions and ion clusters (Alonso, Kousaka, Hashimoto, & Hashimoto, 1997; Otani, Emi, Cho, & Namiki, 1995; Ramamurthi, Strydom, & Hopke, 1990).

In a former investigation (Alonso, Alguacil, & Huang, 2010) an analytical expression was derived for the calculation of the fully developed concentration profile of diffusive particles flowing in a circular tube under laminar flow conditions. The obtained equation, valid for values of the dimensionless diffusion coefficient larger than about 0.2, was successfully

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confirmed by comparison with the results obtained from numerical solution of the convection-diffusion equation. In the present work this equation is used to derive an analytical expression to estimate particle penetration through the tube. It will be shown that the simple proposed equation is in very good agreement with the Gormley–Kennedy equation.

2. Fully developed concentration profile

For the sake of completeness, the derivation of the fully developed concentration profile of diffusive aerosol particles in a laminar flow tube will be briefly summarized here. Further details are to be found in the original publication (Alonso et al., 2010). The starting point is, of course, the convection-diffusion equation, which in the case of negligible axial diffusion can be written as

$$u_x\frac{\partial n}{\partial x}=\beta\left(\frac{\partial^2 n}{\partial r^2}+\frac{1}{r}\frac{\partial n}{\partial r}\right). \tag{1}$$

All the variables in (1) are dimensionless. $n(r,x)$ is the particle number concentration referred to the number concentration at the entrance of the tube, where uniform concentration is assumed; r and x are the radial and axial coordinates, the first referred to the tube radius R, the second to the tube length L; and β is the dimensionless diffusion coefficient, defined as

$$\beta=\frac{DL}{\overline{u_x}R^2}, \tag{2}$$

where D is the diffusion coefficient of the aerosol particle.

The convection-diffusion equation (1) was solved by the method of separation of variables, assuming $n(r,x)=n_{\max}(x)f(r)$. Here, $n_{\max}(x)$ is the maximum value of particle number concentration at section x. This maximum value always takes place at the tube centerline because, due to symmetry, $\partial n/\partial r=0$ at $r=0$. On its part, for the function $f(r)$ a series solution of the form $f(r)=\sum_{s=1}^{\infty}a_{2s}r^{2s}$ was assumed. Note that, again due to symmetry about the tube centerline, the powers of r must be even.

Using the boundary conditions $n=1$ at $x=0$ and $0\le r<1$ (expressing uniform concentration at the tube inlet); $n=0$ at $r=1$ (particles are lost as soon as they reach the wall); and the above mentioned $\partial n/\partial r=0$ at $r=0$, some equations that must be satisfied by the series coefficients a_s could be derived. The expression

$$n=1.477\exp\left(-3.659\beta x\right)\left[1-\tfrac{1}{11}\left(18r^2-9r^4+2r^6\right)\right] \tag{3}$$

was finally obtained. The numerical constants 1.477 and 3.659 were obtained as fitting constants to the numerical solution of (1) for $\beta>0.2$, i.e. for fully developed concentration profile. As shown in the original paper (Alonso et al., 2010), the profile (3) was in fairly good agreement with the numerically calculated profiles, but it deviated appreciably from the numerical solution for values of the dimensionless diffusion coefficient β smaller than about 0.20. In spite of this discrepancy for small values of β, it will be shown below that particle penetration calculated directly from the approximate profile (3) is in excellent agreement with the often verified Gormley–Kennedy equation.

3. Particle penetration

Particle penetration through the tube can be evaluated as

$$P=\frac{\int_0^1 2\pi r u_x(r)n(r,1)dr}{\int_0^1 2\pi r u_x(r)dr}. \tag{4}$$

In the numerator particle number concentration must be evaluated at the tube outlet, $x=1$. Inserting the fully developed concentration profile (3) into (4) and performing the integration, one finds

$$P=0.846\exp\left(-3.659\beta\right). \tag{5}$$

Since (3) is strictly valid for $\beta>0.2$, the validity of Eq. (5) should, in principle, be also restricted to the same range of β values. However, it will be seen below that (5) is actually valid for much smaller values of the dimensionless diffusion coefficient.

4. Comparison with the Gormley–Kennedy equation

The series solution reported by Gormley and Kennedy (1949), written in terms of β, is given by

$$P=0.8191\exp(-3.657\beta)+0.0975\exp(-22.3\beta)+0.0325\exp(-57\beta),\quad \beta\ge 0.0312 \tag{6a}$$