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SHORT COMMUNICATION

PAPR reduction for pilot-aided OFDM systems with the parametric minimum cross-entropy method

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ABSTRACT

This letter considers the selection of the optimal pilot symbol to decrease the peak-to-average power ratio (PAPR) in pilot-aided orthogonal frequency division multiplexing systems. The conventional pilot-aided PAPR reduction technique necessitates an exhaustive search of all combinations of possible pilot symbol configurations, resulting in high computational complexity. To reduce computational complexity while maintaining PAPR reduction performance, we propose the parametric minimum cross-entropy (PMCE) method for determining near-optimal pilot symbols. Simulation results show that performance of the PMCE-based PAPR reduction is very close to that obtained using the exhaustive search algorithm, with low computational complexity. Compared with the standard cross-entropy method, PMCE has the advantages of better PAPR reduction performance at the same computational complexity.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is extensively used in wireless communications because of its high spectral efficiency and robustness against the interference inherent in multipath fading. However, OFDM systems suffer from a large peak-to-average power ratio (PAPR) of transmitted signals, causing signal distortion at the power amplifier output [1]. To reduce the high PAPR, various methods have been presented in the literature, including companding [2], clipping [3], selected mapping [4–6], and partial transmit sequences [7–9].

The pilot symbols applied in OFDM systems for channel estimation and synchronization has recently been recognized as equally applicable to PAPR reduction. The governing principle of this scheme is the reversal of a subset of subcarriers for pilot tones and the selection of one sequence from a set of candidate pilot symbols; thus, the OFDM signal with the selected pilot symbols yields the lowest PAPR [10–12]. In [10,11], the exhaustive search algorithm (ESA) was adopted to identify the optimum pilot symbol from a set of M^{N_p} candidate pilot symbols, where N_p and M are the length of the pilot symbol and number of allowed phases in each pilot tone, respectively. The search complexity of ESA scheme then exponentially increases with the length of pilot symbols. The techniques proposed for reducing complexity accomplish such reduction at the cost of PAPR performance [12].

In this letter, we propose the application of parametric minimum cross-entropy (PMCE) [13] to identify the optimal pilot symbol required to reduce the PAPR of pilot-aided OFDM systems. Simulation results demonstrate that the PMCE-based PAPR reduction exhibits a performance that is very close to that obtained using the ESA scheme; however, the computational complexity of the latter is considerably lower than that of the ESA scheme. In addition, the proposed PAPR reduction outperforms cross-entropy (CE) methods [14] under the same level of complexity.

The remainder of this letter is organized as follows. Section 2 describes the signal models for pilot-aided OFDM systems. The proposed PMCE algorithm for the design of optimal pilot symbols is developed in Section 3. Section 2 demonstrates the simulation results. Finally, Section 6 ends with some concluding remarks.

2. System models

For an OFDM system with N subcarriers, an inverse discrete Fourier transform (IDFT) is adopted on frequency domain data vector $\mathbf{X}$ to obtain time-domain signal vector $\mathbf{x}$, where the nth element of $\mathbf{x}$ is expressed as

$$x_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_k e^{j2\pi kn/LN}, \quad n = 0, 1, \ldots, LN-1, \tag{1}$$

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where X_k is the kth element of $\mathbf{X}$ and L represents the oversampling factor adopted to provide a sufficiently accurate PAPR estimate. The PAPR of the OFDM signal is defined as

$$\mathrm{PAPR}(\mathbf{x}) = \frac{\max_{0\le n\le LN-1}|x_n|^2}{E[|x_n|^2]}, \tag{2}$$

where $\max|x_n|^2$ denotes the maximum peak power, and $E[|x_n|^2]$ denotes the average power.

In pilot-aided OFDM systems, frequency domain data vector $\mathbf{X}$ can be divided into two components, namely,

$$X_k = \begin{cases} D_k, & k \in \Upsilon^{\perp} \\ S_k, & k \in \Upsilon, \end{cases} \tag{3}$$

where Υ denotes the set of pilot tone indices, and $\Upsilon^{\perp}$ denotes the complement of Υ. In addition, S_k and D_k are the kth elements of frequency domain pilot symbol $\mathbf{S}$ and data symbols $\mathbf{D}$, respectively. The nth element of the time-domain signal vector $\mathbf{x}$ is given by

$$x_n = s_n + d_n, \tag{4}$$

where d_n and s_n are the nth elements of time-domain data symbols and pilot symbols, respectively. Fig. 1 depicts the idea of pilot-aided PAPR reduction scheme in OFDM systems. The governing principle of this scheme is the reversal of a subset of subcarriers for pilot tones and the selection of one sequence from a set of candidate pilot symbols; thus, the OFDM signal with the selected pilot symbols yields the lowest PAPR. $\mathbf{S}_u$ denotes the uth frequency-domain pilot symbol in vector form and U is the number of candidate signals. The pilot-aided PAPR reduction scheme primarily involves the identification of the pilot symbol that minimizes the PAPR, i.e.,

$$\begin{aligned} \mathrm{PAPR}(\tilde{\mathbf{x}}) &= \min_{\mathbf{S}_u}\{\mathrm{PAPR}(\mathbf{x}_u)\}, \quad 0 \le u \le U-1 \\ &= \min_{\mathbf{S}_u}\left\{\frac{\max_{0\le n\le LN-1}|x_{n,u}|^2}{E[|x_{n,u}|^2]}\right\}, \end{aligned} \tag{5}$$

where $x_{n,u} = s_{n,u} + d_n$ is the uth time-domain candidate signal comprising of the data symbol d_n and the uth pilot symbol $s_{n,u}$. Consequently, the optimal pilot symbol design for minimizing the PAPR of an OFDM signal is related to the combinatorial optimization problem, i.e.,

$$\text{minimize} \quad \mathrm{PAPR}(\mathbf{Q}(\mathbf{D}+\mathbf{S})) \tag{6}$$

$$\text{subject to} \quad S_k \in \{A_k e^{j\theta_k}\}^{N_p}, k \in \Upsilon, \theta_k \in \{0 \sim 2\pi\}$$

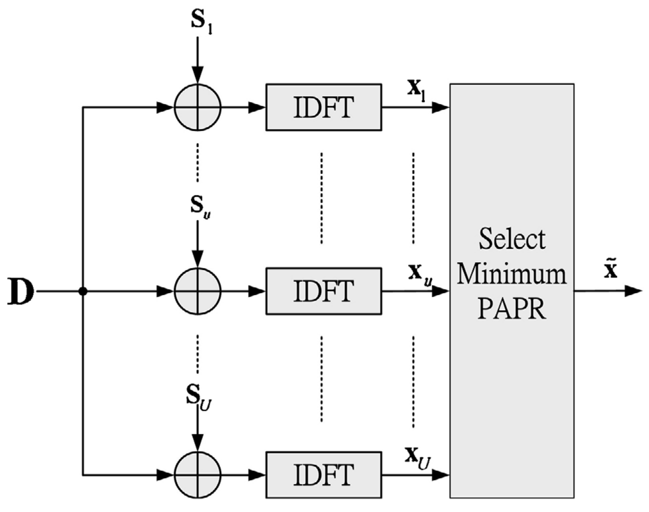

Fig. 1. Block diagram of pilot-aided PAPR reduction scheme in OFDM systems.

where $\mathbf{Q}$ denotes the IDFT matrix, N_p is the length of the pilot symbol. In addition, A_k and θ_k is the amplitude and phase of kth pilot signal, respectively. The optimal solution of (5) is to perform ESA over all possible parameter combinations of pilot symbols. Same-amplitude pilot symbols are optimal for channel estimation, but the optimal solution of (5) remains a difficult problem with the search complexity of M^{N_p}, where M is the number of allowed phases in each pilot tone; that is, $\theta_k = 2\pi l/M$, $l = 0, 1, \ldots, M-1$. Thus, ESA possess exponential search complexity with N_p.

3. Pilot-aided PAPR reduction using parametric minimum cross-entropy (PMCE)

The parametric minimum cross-entropy (PMCE) method which was originally proposed by [13] for solving combinatorial optimization problems is adopted to determine the near-optimum pilot signal. It can be summarized as follows:

(1) It generates random data samples according to a specified probability distribution function.
(2) It updates the parameters of probability distribution to amend samples in the succeeding iteration.

The detailed definition and expressions of the PMCE method and its applications are provided in [13]. In order to employ the PMCE method to find the pilot sequences that minimize the PAPR in pilot-aided OFDM systems, we have to define the fitness function for the proposed PMCE scheme. The fitness function can be expressed as

$$F(\mathbf{S}) = \mathrm{PAPR}(\mathbf{Q}(\mathbf{D}+\mathbf{S})). \tag{7}$$

In this letter, the optimization problem in the proposed PMCE method is the minimization of the fitness function. The selection of the pilot symbols is generally limited to a set with a finite number of candidate pilot symbols. In this letter, phases $\theta_k = \{0, \pi\}$ (i.e., $M = 2$) and amplitude $|S_k| = A_k = 1$ are chosen. Thus, the minimization of the fitness function is transformed into the following equations:

$$\text{minimize} \quad F(\mathbf{S}) = \mathrm{PAPR}(\mathbf{Q}(\mathbf{D}+\mathbf{S})) \tag{8}$$

$$\text{subject to} \quad S_k \in \{1, -1\}^{N_p}, k \in \Upsilon$$

Notably, minimizing the fitness function defined by (8) necessitates 2^{N_p} candidate pilot symbols. Inspired by the efficiency of the PMCE method in solving the combinatorial optimization problem, we propose the PMCE method to determine the near-optimal pilot symbol. The procedure of the proposed PMCE-based PAPR reduction is described as follows:

Step (1) Iteration counter $i = 1$ is set and probability $\mathbf{p}^0 = \{p_m^0\}_{m=0}^{N_p-1}$ is initialized with $p_m^0 = 0.5$ for all m, where p_m^0 denotes the probability of the mth element of the pilot symbol.

Step (2) Each element of $\mathbf{S}$ is modeled as an independent Bernoulli random variable with probability mass function (PMF) $P(S_k = 1) = p_k$ and $P(S_k = 0) = 1 - p_k$ for $k = 0, 1, \ldots, N_p - 1$. The PMF is defined as

$$f(\mathbf{S};\mathbf{p}) = \prod_{k=0}^{N_p-1} p_k^{S_k} \cdot (1-p_k)^{1-S_k} \tag{9}$$

Using the PMF $f(\mathbf{S};\mathbf{p}^{i-1})$ to generate K random samples $\mathbf{S}_1^{i-1}, \mathbf{S}_2^{i-1}, \ldots, \mathbf{S}_K^{i-1}$ and then calculating the fitness function according to (8) yields $F(\mathbf{S}_1^{i-1}), F(\mathbf{S}_2^{i-1}), \ldots, F(\mathbf{S}_K^{i-1})$.