Autocratic strategies for alternating games

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A B S T R A C T

Repeated games have a long tradition in the behavioral sciences and evolutionary biology. Recently, strategies were discovered that permit an unprecedented level of control over repeated interactions by enabling a player to unilaterally enforce linear constraints on payoffs. Here, we extend this theory of "zero-determinant" (or, more generally, "autocratic") strategies to alternating games, which are often biologically more relevant than traditional synchronous games. Alternating games naturally result in asymmetries between players because the first move matters or because players might not move with equal probabilities. In a strictly-alternating game with two players, X and Y, we give conditions for the existence of autocratic strategies for player X when (i) X moves first and (ii) Y moves first. Furthermore, we show that autocratic strategies exist even for (iii) games with randomly-alternating moves. Particularly important categories of autocratic strategies are extortionate and generous strategies, which enforce unfavorable and favorable outcomes for the opponent, respectively. We illustrate these strategies using the continuous Donation Game, in which a player pays a cost to provide a benefit to the opponent according to a continuous cooperative investment level. Asymmetries due to alternating moves could easily arise from dominance hierarchies, and we show that they can endow subordinate players with more autocratic strategies than dominant players.

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1. Introduction

Repeated games, and, in particular, the repeated Prisoner's Dilemma, have been used extensively to study the reciprocation of cooperative behaviors in social dilemmas (Trivers, 1971; Axelrod and Hamilton, 1981; Axelrod, 1984; Nowak, 2006). These games traditionally involve a sequence of interactions in which two players act simultaneously (or, at least without knowing the opponent's move) and condition their decisions on the history of their previous encounters. Even though such synchronized decisions seem often contrived in realistic social interactions, the biologically more realistic and relevant scenario with asynchronous interactions has received surprisingly little attention. In asynchronous games, players take turns and alternate moves in either a strict or random fashion (Nowak and Sigmund, 1994; Wedekind and Milinski, 1996).

A classic example of an asynchronous game with alternating moves is blood donation in vampire bats (Wilkinson, 1984). When a well-fed bat donates blood to a hungry fellow, the recipient has the opportunity to return the favor at a later time. Similarly, social grooming between two primates is not always performed simultaneously; instead, one animal grooms another, who then has the opportunity to reciprocate in the future (Muroyama, 1991). On a smaller scale, the biosynthesis of iron-scavenging compounds by microorganisms through quorum sensing can result in asynchronous responses to fellow "players" in the population (Stintzi et al., 1998; Miller and Bassler, 2001; Iliopoulos et al., 2010). Even for interactions that appear to involve simultaneous decisions, such as in acts of predator inspection by fish (Milinski, 1987), it remains difficult to rule out that these interactions are not instead based on rapid, non-synchronous decisions (Frean, 1994).

The iterated Prisoner's Dilemma game, which involves a choice to either cooperate, C, or defect, D, in each round, has played a central role in the study of reciprocal altruism (Axelrod and Hamilton, 1981; Axelrod, 1984; Nowak, 2006). Rather unexpectedly, after decades of intense study of iterated games, Press and Dyson (2012) showed that a player can unilaterally enforce linear payoff relationships in synchronous games. For example, if π_X and π_Y are the expected payoffs to players X and Y, respectively, and $\chi \geqslant 1$ is an extortion factor, then player X can ensure that $\pi_X = \chi \pi_Y$, regardless of the strategy of player Y.

A

player X	C		D		D		
player Y		D		C		C	
round	0	1	2	3	4	5	$\cdots$
	↓	↓	↓	↓	↓	↓	
(X's payoff, Y's payoff)	$(-c,b)$	$(0,0)$	$(0,0)$	$(b,-c)$	$(0,0)$	$(b,-c)$	

B

player X		C		D		D	
player Y	D		C		C		
round	0	1	2	3	4	5	$\cdots$
	↓	↓	↓	↓	↓	↓	
(X's payoff, Y's payoff)	$(0,0)$	$(-c,b)$	$(b,-c)$	$(0,0)$	$(b,-c)$	$(0,0)$	

C

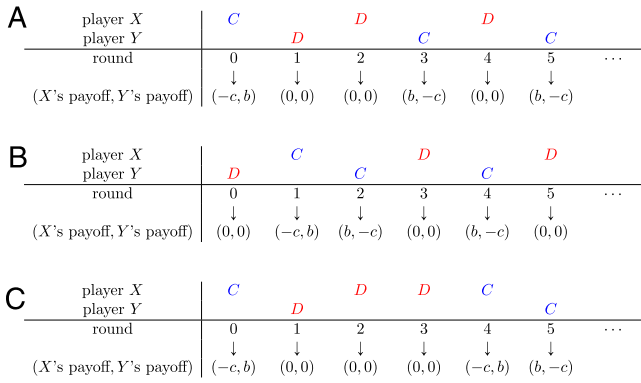| player X	C		D	D	C		
player Y		D				C	
round	0	1	2	3	4	5	$\cdots$
	↓	↓	↓	↓	↓	↓	
(X's payoff, Y's payoff)	$(-c,b)$	$(0,0)$	$(0,0)$	$(0,0)$	$(-c,b)$	$(b,-c)$	

Fig. 1. Three types of interactions in the alternating Donation Game: (A) strictly-alternating game in which player X moves first; (B) strictly-alternating game in which player Y moves first; and (C) randomly-alternating game in which, in each round, player X moves with probability ω_X and player Y with probability $1-\omega_X$. For each type of alternating game, a player moves either C or D (cooperate or defect) in each round and both players receive a payoff from this move. Unlike in strictly-alternating games, (A) and (B), a player might move several times in a row in a randomly-alternating game, (C).

Moreover, such linear relationships may be enforced using merely memory-one strategies, which condition the next move on the outcome of just the previous round.

The discovery of these so-called "zero-determinant" strategies triggered a flurry of follow-up studies. Most notably, from an evolutionary perspective, extortionate strategies fare poorly (Hilbe et al., 2013) but can be stable provided that extortioners recognize one another (Adami and Hintze, 2013). However, generous counterparts of extortionate strategies perform much better in evolving populations (Stewart and Plotkin, 2012, 2013) and constitute Nash equilibria for the repeated Prisoner's Dilemma (Hilbe et al., 2015) (but generally only if there are just two discrete levels of cooperation (McAvoy and Hauert, 2016)). Against humans, extortionate strategies typically underperform generous strategies when the extortioner is also a human (Hilbe et al., 2014) but can outperform generous strategies when the extortioner is a computer (Wang et al., 2016). Thus, for the settings in which zero-determinant strategies are known to exist, their performance is sensitive to the context in which they arise. Our focus here is on extending these strategies further into the domain of alternating interactions from a classical, non-evolutionary viewpoint. In particular, we establish the existence of zero-determinant strategies for several types of alternating interactions between two players.

Recently, autocratic strategies were introduced as a generalization of zero-determinant strategies to simultaneous games with arbitrary action spaces (McAvoy and Hauert, 2016). An autocratic strategy for player X is any strategy that, for some constants α, β, and γ (not all zero), enforces the linear relationship

$$\alpha \pi_X + \beta \pi_Y + \gamma = 0 \tag{1}$$

on expected payoffs every strategy of player Y. Here, we consider autocratic strategies in alternating games. In a strictly-alternating game, one player moves first (either X or Y) and waits for the opponent's response before moving again. This process then repeats, with each player moving only after the opponent moved (see Fig. 1(A), (B)). In contrast, in a randomly-alternating game, the player who moves in each round is chosen stochastically: at each time step, X moves with probability ω_X and Y moves with probability $1 - \omega_X$ for some $0 \leqslant \omega_X \leqslant 1$ (see Fig. 1(C)). Note that only for $\omega_X = 1/2$ is it the case that both players move, on average, equally often.

Previous studies of zero-determinant strategies have focused on enforcing linear payoff relationships using conditional responses with short memories. A player using a memory-one strategy determines his or her response (stochastically) based on the outcome of just the previous round. Although strategies with longer memory length have been shown to help establish cooperation (Hauert and Schuster, 1997; Stewart and Plotkin, 2016), they are not always reliably implemented in players with limited memory capacity (including humans) (Milinski and Wedekind, 1998; Stevens et al., 2011; Baek et al., 2016). Here, we follow the tradition of concentrating on shorter-memory strategies. In particular, we establish the existence of memory-one autocratic strategies for alternating games and give several simple examples that enforce linear payoff relationships for every strategy of the opponent (even those with unlimited memory).

In the classical Donation Game (Sigmund, 2010), a player either (i) cooperates and donates b to the opponent at a cost of c or (ii) defects and donates nothing and pays no cost, which yields the payoff matrix

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{cc} C & D \\ \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array} \tag{2}$$

and represents an instance of the Prisoner's Dilemma provided that benefits exceed the costs, $b > c > 0$. The continuous Donation Game extends this binary action space and allows for a continuous range of cooperation levels (Killingback et al., 1999; Wahl and Nowak, 1999a,b; Killingback and Doebeli, 2002). An action in this game is an investment level, s, taken from an interval, $[0, K]$, where K indicates an upper bound on investments. Based on its investment level, s, a player then pays a cost of $c(s)$ to donate $b(s)$ to the opponent where $b(s)$ and $c(s)$ are continuous non-decreasing functions with $b(s) > c(s) > 0$ for $s > 0$ and $b(0) = c(0) = 0$; an investment of zero corresponds to defection, which neither generates benefits nor incurs costs (Killingback and Doebeli, 2002). Biologically-relevant interpretations of continuous investment levels (as well as alternating moves) include (i) the effort expended in social grooming and ectoparasite removal by primates (Dunbar, 1991); (ii) the quantity of blood donated by one vampire bat to another (Wilkinson, 1984); (iii) the amount of iron-binding agents (siderophores) produced by bacterial parasites (West and Buckling, 2003); and (iv) the honesty level of a (human) party involved in a trade agreement (Verhoeff, 1998).

In alternating games, the assignment of payoffs to players deserves closer inspection (Hauert and Schuster, 1998). Here, we focus on alternating games in which both players obtain payoffs after every move (like in the continuous Donation Game) (see Fig. 1; Nowak and Sigmund, 1994). Alternatively, payoffs could result from every pair of moves rather than every individual move (Frean, 1994). While it is possible to construct a theory of autocratic strategies for strictly-alternating games in either setting, it becomes difficult to even define payoffs in the latter setup for randomly-alternating games because either player can move several times in a row (see Fig. 1(C)). Therefore, we follow Nowak and Sigmund (1994) in order to include the particularly relevant and intriguing case of randomly-alternating games.

Randomly-alternating games seem more relevant for modeling biological interactions because often strict alternation cannot be maintained or enforced, or the players find themselves in different roles, which translate into different propensities to move. To accommodate these situations, we consider, a class of randomly-alternating games in which the probability that player X moves in a given round, ω_X, is not necessarily $1/2$. Any other value of ω_X results in asymmetric interactions – even if the payoffs in each encounter are symmetric – simply because one player moves more often than the other. For example, dominance hierarchies in primates naturally result in asymmetric behavioral patterns (Mehlman and Chapais, 1988; Lazaro-Perea et al., 2004; Newton-Fisher and Lee, 2011). In male chimpanzees, dominance hierarchies require smaller, subordinate chimpanzees to groom