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Assessing network robustness under SIS epidemics: The relationship between epidemic threshold and viral conductance

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ABSTRACT

Telecommunication networks, as well as other network types, are critical infrastructures where any service disruption has a notable impact on individuals. Hence, studying network dynamics under failures or attacks is of paramount importance. In this paper, we assess the robustness of networks with respect to the spread of Susceptible-Infected-Susceptible (SIS) epidemics, using the N-Intertwined Mean-Field Approximation (NIMFA). A classical robustness metric is the NIMFA epidemic threshold, which is inversely proportional to the largest eigenvalue of the adjacency matrix, also called the spectral radius. Besides the NIMFA epidemic threshold, the viral conductance has been proposed as a measure incorporating the average fraction of infected nodes in the steady state for all possible effective infection rates. In general, the viral conductance provides more information about the network's behavior with respect to virus spreading, however, the full picture is not always necessary. The aim of this paper is to understand when the spectral radius is adequate for reflecting robustness. By analyzing the relationship between spectral radius and viral conductance in several graph classes, we show that the two metrics are highly correlated. We thus conclude that the spectral radius is sufficient to compare the robustness of networks belonging to the same class.

1. Introduction

Much effort has been devoted in the analysis of the spread of epidemics, mainly due to the expected outbreak of new lethal biological viruses affecting individuals and the increasing threats from cybercrime. In November 2014, for example, the Ebola epidemic caused 9596 laboratory-confirmed cases of individuals infected by the virus with 5459 total deaths, becoming the largest epidemic in history affecting multiple countries in West Africa. In addition to epidemic spread among individuals, there are other important scenarios like telecommunication systems, power grids and transportation networks, where the theory of the spread of epidemics can be applied to characterize their vulnerability. E-mail worms, computer viruses, the propagation of failures in power grids, and, more generally, the spread of information and epidemic dissemination/routing in ah-hoc and peer-to-peer networks are just some examples of scenarios, where studying the spread of epidemics is crucial for maintaining high levels of robustness. Assessing network robustness not only allows us to compare the robustness among different network topologies, but also gives insights in the design of future networks to mitigate the spreading of a virus or cascading failures, or maximize information diffusion (e.g. news, rumors, brand awareness, marketing of new products, etc.).

In this paper, we focus on the spread of epidemics on telecommunication networks. Network operators are mainly interested in understanding (a) how much robust their network is compared to other networks and (b) how to protect or modify their infrastructure for improving its robustness minimizing costs. The aim of this paper is to answer the first question focusing on the spread of epidemics described through a Susceptible-Infected-Susceptible (SIS) model [1]. This model, which arose in mathematical biology, and its variants, are often used for the spread of viruses and malwares in computer networks [2–4], and mobile social opportunistic networks [5], epidemic information dissemination in unreliable distributed systems like P2P and ad-hoc networks [6], cascading failures on BGP networks [7] and power grids [8], smartphone malware propagation [9], and epidemic spreading in

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wireless sensor networks [10]. A network is represented by an undirected graph $G(N, L)$, characterized by a symmetric adjacency matrix A in which the element $a_{ij} = a_{ji} = 1$ if there is a link between nodes i and j, otherwise $a_{ij} = 0$. The infectious state of a node i in G is specified by a Bernoulli random variable $X_i \in \{0, 1\} : X_i = 0$ for a *healthy* node and $X_i = 1$ for an *infected* node. A node i at time t can be in the infected state, with probability $v_i(t) = \Pr[X_i(t) = 1]$, or in the healthy state, with probability $1 - v_i(t)$. Both the arrival of an infection over a link and the curing of an infected node i are assumed to be independent Poisson processes with rates β and δ, respectively, and the effective spreading rate is defined as $\tau = \frac{\beta}{\delta}$. This SIS model can be expressed exactly in terms of a continuous-time Markov chain with 2^N states as shown in [11]. Since the exact solution $v_i(t)_{1 \leq i \leq N}$ for any network is intractable, several approximations of the SIS model have been proposed. In this paper, we focus on the N-Intertwined Mean-Field Approximation (NIMFA) [11] that was earlier considered in discrete time in [12] and in [13], whose paper was later improved in [14]. NIMFA has been shown in [15] to be better than the widely used heterogeneous mean-field model of Pastor-Satorras and Vespignani [16].

A remarkable property of the SIS model is the existence of a phase transition [16] when the effective spreading rate approaches an epidemic threshold τ_c: if $\tau > \tau_c$, the infection becomes persistent, while if $\tau < \tau_c$, the virus dies out and the network is virus-free. Many authors (see, e.g., [2,17–19]) mentioned the existence of an *epidemic threshold*, however, the determination of this epidemic threshold is a long-standing open problem and a major contribution of NIMFA is the lower bound $\tau_c^{(1)} = \frac{1}{\lambda_1} \leq \tau_c$, where $\tau_c^{(1)}$ is the NIMFA epidemic threshold and λ_1 is the largest eigenvalue of the network adjacency matrix A, also called the spectral radius. The NIMFA epidemic threshold has been used as a measure for network robustness [20]. Recently, the *viral conductance*, introduced in [21,22] and analyzed in depth in [23], was proposed as a robustness measure considering both the value of the epidemic threshold and the number of infected nodes at the steady state above the threshold. The viral conductance measures the integrated effect over all possible viral infection strengths on the network's infectious vulnerability and is shown to be a better robustness measure than the epidemic threshold, considering that viral conductance provides more information about the system's behavior [22].

In this paper, we present an extensive analysis on the relationship between the viral conductance and the spectral radius[1] in telecommunication networks. We chose to model the relationship between epidemic threshold and viral conductance for the SIS model since these two robustness metrics have been defined for this epidemic model. For the SIR model, for example, which is variant of the SIS model having a third state where a node can be recovered (R), there exist similar bounds for the epidemic threshold [24] but there is not yet an expression for the viral conductance. This is the reason why we start analyzing the SIS model and leave the study of the relationship between the two robustness metrics in other epidemic models to future investigations. The aim of this analysis is to understand if both the two metrics are necessary for characterizing the robustness of a network with respect to the spreading of a virus. In particular, we have chosen to test a set of topologies to model the most widely diffused telecommunication networks (e.g. complete bipartite graphs for core telecommunication networks, Erdős–Rényi random graphs [25] for peer-to-peer and ad hoc networks, Watts–Strogatz small-world graphs [26] for mobile contact networks, Bárabasi–Albert [27] scale-free graphs for social networks, etc.) and a set of real-world Internet backbones. The contributions of this paper can be summarized as follows:

- we derive easy to use upper and lower bounds for the viral conductance as a function of the spectral radius;
- via examples, we show cases where both the viral conductance and the spectral radius correctly compare the robustness between two networks, and cases where the spectral radius fails to assess robustness;
- we consider several graph classes representative of different types of networks and derive the relationship between the viral conductance and the spectral radius analytically, where possible, or through a correlation analysis;
- we show that the two metrics are highly correlated and hence, the information provided by the spectral radius is adequate for comparing the robustness of networks belonging to the same graph class.

The rest of this paper is organized as follows. Section 2 reviews the literature and related works. Section 3 reviews NIMFA. The viral conductance measure is described in Section 4. Section 5 compares the viral conductance and the epidemic threshold through case studies. Section 6 and 7 analyze the relationship between the viral conductance and the spectral radius in various graph classes. Conclusions are summarized in Section 8.

2. Related works

The spread of information or viruses, or the propagation of failures in communication networks are similarly described as the virus spread in a biological population [28]. Biological epidemic models were initiated by Kephart and White [2], for example, to describe the spread of viruses in computer networks. In classical epidemiology, epidemic models such as [1,29] introduced the basic reproductive number R_0 representing the average number of infections due to a single infected case in the population. For $R_0 < 1$, the epidemic dies out without causing an outbreak while for $R_0 > 1$, the epidemic spreads in the population. The basic reproductive number R_0 bears some resemblance with the epidemic threshold [16] defined in complex network theory for SIS epidemic models using the heterogeneous mean-field approach. However, the basic reproductive number R_0 does not contain any information about the underlying contact network. Since the structure of networks is rich and complex, R_0 is inadequate for assessing the threshold behavior of an outbreak in most networks.

In the literature dealing with the epidemic threshold in networks, Wang et al. [13] proposed a discrete-time epidemic model to predict the infection size within a population suggesting that the epidemic threshold equals the reciprocal of the spectral radius of the adjacency matrix A. In another work, Van Mieghem et al. [11] studied SIS epidemics as a continuous-time Markov chain and introduced NIMFA, whose epidemic threshold was proved to be equal to the inverse of the spectral radius of A. In [30], an heterogeneous version of NIMFA was used to design a strategy for controlling an epidemic outbreak in an arbitrary contact network by distributing vaccination resources throughout the network. More recently, NIMFA has been extended to model the SIS epidemic spread in networks of individuals partitioned into communities [31,32], where the infection rate by which an individual infects individuals in its own community is different from the inter-community infection rate. In another recent work [33] , NIMFA has been used to design resilient and secure networks for cyber attack-induced cascading link failures in critical infrastructures.

The epidemic threshold was initially used to assess network robustness with respect to the spread of epidemics [20]: the larger

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[1] For the easier readability of the results obtained in this work, we consider the reciprocal of the NIMFA epidemic threshold (i.e. the spectral radius) when analyzing in deep the relationship between the two robustness metrics.