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A peridynamic model of flow in porous media

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ABSTRACT

This paper presents a nonlocal, derivative free model for transient flow in unsaturated, heterogeneous, and anisotropic soils. The formulation is based on the peridynamic model for solid mechanics. In the proposed model, flow and changes in moisture content are driven by pairwise interactions with other points across finite distances, and are expressed as functional integrals of the hydraulic potential field. Peridynamic expressions of the rate of change in moisture content, moisture flux, and flow power are derived, as are relationships between the peridynamic and the classic hydraulic conductivities; in addition, the model is validated. The absence of spacial derivatives makes the model a good candidate for flow simulations in fractured soils and lends itself to coupling with peridynamic mechanical models for simulating crack formation triggered by shrinkage and swelling, and assessing their potential impact on a wide range of processes, such as infiltration, contaminant transport, and slope stability.

1. Introduction

Naturally occurring soils, especially fine-textured ones, exhibit shrinking and swelling behavior [1–3]. These soils tend to swell when their moisture content increases, and shrink when it decreases. At the field scale, this behavior leads to tensile stresses that may exceed the soil's failure limit and trigger the formation and evolution of cracks during drying phases. Cracks may in turn close during infiltration phases when the soil becomes wetter and swells [4–6] giving them a dynamic nature leading to highly nonlinear responses. These desiccation cracks have a length scale of ten to a hundred centimeters and their effect on the hydraulic properties of the soil is not captured by standard laboratory tests using a Representative Elementary Volume (REV) with a length scale of a few centimeters.

Desiccation cracking has a wide spectrum of environmental, agricultural, and hydrological impacts. The movement of moisture and solutes into and within the soil increases due to the presence of these cracks that act as preferential pathways for rapid water movement to deeper layers [7–11]. This rapid movement may lower the effectiveness of irrigation [12] and causes fast seepage of nutrients and pesticides away from the plants into deeper layers reducing the contaminants' residence time in the unsaturated zone where they are usually absorbed by the plants and degraded by bacteria, and increasing the probability of ground water and/or surface water contamination, depending on the relief. In addition, desiccation cracks can have a dramatic effect on processes of surface water movement and flood dynamics by altering the partitioning of rainfall between infiltration and runoff, which is an important issue to consider when modeling and forecasting flood events.

Desiccation cracks also have engineering and geotechnical impacts with potentially very serious environmental and public safety repercussions. For example, desiccation cracks developing at the surface of a slope may trigger the onset of a landslide. If they develop in the core of an earth dam, cracks act as preferential moisture flow paths, increasing the moisture content of the dam and, with it, the pore water pressure which eventually leads to its failure [13]. Clay barriers used in landfills and nuclear waste disposal sites are also subject to desiccation cracking which reduces the barrier's containment effectiveness [14,15].

In this paper, we present a peridynamic model for transient moisture flow in unsaturated, heterogeneous, and anisotropic soils. The model is an alternative to the classic Richard's equation and is based on Silling's reformulation of the theory of elasticity for solid mechanics [16,17]. In the proposed model, we replace the classic, local, continuum mechanics formulation by a nonlocal integral functional. The model is free of spacial derivatives, and the flow is driven by the hydraulic potential field instead of the gradient of the hydraulic potential field. Katiyar et al. [18] have derived similar peridynamic formulations for saturated steady state flow.

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Due to the lack of spacial derivatives, this model is capable of handling the spurious formation of cracks, which translate into points of singularities in the parameter and hydraulic potential fields, within the simulation domain without failing. This would allow us to couple the derived model with a peridynamic mechanical model and simulate the formation of desiccation cracks and their dynamics and assess the potential of such an approach on evaluating their impact on flow and solute transport. This coupling is however the subject of a subsequent paper.

We would like to point out that the nonlocal aspect of the proposed model is related to the mechanism of state change in the domain. In classic nonlocal formulations [19–25], the new value of a state is the one with the maximum likelihood and the change is driven by some statistical measure of the gradient of the driving field within the surrounding region. On the other hand, in peridynamic models such as this one, the change of state at a point is driven by the influence of the value of some field at points that are at some finite distance away.

We will start by presenting the peridynamic model concept and derive the peridynamic expression for the rate of change of moisture content. We will then derive the peridynamic equations of flow power dissipation and moisture flux, which we will use in deriving the relationship between the peridynamic hydraulic conductivity density and the classic hydraulic conductivity for unsaturated, homogeneous, heterogeneous, isotropic, and anisotropic soils. We will also show that the peridynamic model equations of moisture flow and flux converge to the classic Richard's and Darcy's equations at the limit of vanishing horizon. This will be followed by a presentation of the numerical implementation and validation of the model in one and two dimensions.

2. Peridynamic flow model

Consider the homogeneous and isotropic body of soil Ω in Fig. 1, where each point $\mathbf{x}$ in Ω represents a differential volume dV_x [L^3], and is at some total hydraulic potential $H(\mathbf{x})$ [L]. Suppose the change in moisture content at every point $\mathbf{x}$ in Ω is driven by pairwise interactions with all other points $\mathbf{x}'$ in Ω despite the finite distance separating each pair points.

Suppose also that these pairwise interactions are equivalent to a one dimensional resistive pipe that acts as a conduit and does not store any moisture, that we will call peripipe, and that each peripipe has a property called the peridynamic hydraulic conductance density, $C(\mathbf{x},\mathbf{x}')$ [$T^{-1}L^{-4}$], which is equal to the volume of moisture that will flow per second in peripipe $\mathbf{xx}'$ per unit hydraulic potential difference, per unit volume of $\mathbf{x}$, and per unit volume of $\mathbf{x}'$.

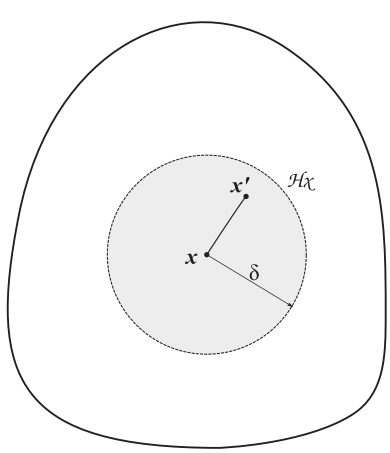

Fig. 1. Peridynamic medium representation. Point $\mathbf{x}$ is influenced by all points within its horizon. $\mathcal{H}_x$ is the horizon of $\mathbf{x}$, δ is the radius of the horizon.

We can now define the pairwise interaction which we will call the peridynamic flow density function $J(\mathbf{x},\mathbf{x}')$ [$T^{-1}L^{-3}$], as the rate of moisture flow from point $\mathbf{x}'$ to point $\mathbf{x}$ per unit volume of $\mathbf{x}$ per unit volume of $\mathbf{x}'$:

$$J(\mathbf{x},\mathbf{x}') = C(\mathbf{x},\mathbf{x}')[H(\mathbf{x}') - H(\mathbf{x})], \tag{1}$$

where the peripipe conductance $C(\cdot)$ is calculated from the peridynamic hydraulic conductivity density, $\kappa(\mathbf{x}',\mathbf{x})$ [$T^{-1}L^{-3}$], an intrinsic material property which is not equal to, but can be related to the classic hydraulic conductivity K [LT^{-1}].

$$C(\mathbf{x},\mathbf{x}') = \frac{\kappa(\mathbf{x}',\mathbf{x})}{\|\mathbf{xx}'\|}. \tag{2}$$

The change of moisture stored at $\mathbf{x}$, and that of point $\mathbf{x}'$ mediated by peripipe $\mathbf{xx}', \Delta V_m(\mathbf{x},\mathbf{x}')$ [L^3], and $\Delta V_m(\mathbf{x}',\mathbf{x})$ [L^3] respectively are given by:

$$\Delta V_m(\mathbf{x},\mathbf{x}') = \kappa(\mathbf{xx}')\frac{[H(\mathbf{x}') - H(\mathbf{x})]}{\|\mathbf{xx}'\|}dV_{x'}dV_x, \tag{3}$$

$$\Delta V_m(\mathbf{x}',\mathbf{x}) = \kappa(\mathbf{x}'\mathbf{x})\frac{[H(\mathbf{x}) - H(\mathbf{x}')]}{\|\mathbf{x}'\mathbf{x}\|}dV_x dV_{x'}. \tag{4}$$

Because peripipes do not store any moisture, and due to conservation of mass we have $\Delta V_m(\mathbf{x},\mathbf{x}') = -\Delta V_m(\mathbf{x}',\mathbf{x})$, and because $\|\mathbf{xx}'\| = \|\mathbf{x}'\mathbf{x}\|$, we get the following restriction on $\kappa(\cdot)$:

$$\kappa(\mathbf{x}',\mathbf{x}) = \kappa(\mathbf{x},\mathbf{x}'). \tag{5}$$

Using Eq. (1), the total change in volumetric moisture content at any point $\mathbf{x}$ in Ω due to its interaction with all other points $\mathbf{x}'$ in Ω, in addition to external sources, or sinks, of moisture at point $\mathbf{x}, S(x)$ [T^{-1}] is given by the following functional integral:

$$\frac{\partial\theta}{\partial t}(\mathbf{x}) = \int_\Omega \kappa(\mathbf{x},\mathbf{x}')\frac{[H(\mathbf{x}') - H(\mathbf{x})]}{\|\mathbf{xx}'\|}dV_{x'} + S(x), \quad [T^{-1}]. \tag{6}$$

Eq. (6) is the peridynamic equation of flow for unsaturated porous media. Note that there are no restrictions on $C(\cdot,\cdot)$ beyond the one stated in Eq. (5) and being integrable. Integrating this equation over the entire domain Ω, we get the total change of moisture in the domain:

$$\int_\Omega \frac{\partial\theta}{\partial t}(\mathbf{x})dV_x = \int_\Omega\int_\Omega \kappa(\mathbf{x},\mathbf{x}')\frac{[H(\mathbf{x}') - H(\mathbf{x})]}{\|\mathbf{xx}'\|}dV_{x'}dV_x + \int_\Omega S(x)dV_x, \quad [L^3]. \tag{7}$$

Rewriting the first integral in the right hand side as follows:

$$\int_\Omega\int_\Omega \frac{\kappa(\mathbf{x}',\mathbf{x})H(\mathbf{x}')}{\|\mathbf{xx}'\|}dV_{x'}dV_x - \int_\Omega\int_\Omega \frac{\kappa(\mathbf{x}',\mathbf{x})H(\mathbf{x})}{\|\mathbf{xx}'\|}dV_{x'}dV_x, \tag{8}$$

and switching the variables x and x' in the second integral and reversing the integration order while keeping in mind the restriction on $C(\cdot,\cdot)$ (Eq. (5)), we realize that Eq. (8) evaluates to zero, and Eq. (7) becomes:

$$\int_\Omega \frac{\partial\theta}{\partial t}(\mathbf{x})dV_x = \int_\Omega S(x)dV_x. \tag{9}$$

Eq. (9) is a statement of conservation of mass in the domain and it states that the total change in moisture content in the domain Ω is equal to the total amount of moisture added, or removed from external sources.

We will now introduce an additional property of a peridynamic medium, which is that any two points $\mathbf{x}$ and $\mathbf{x}'$ separated by a distance greater than a maximum distance δ are too far apart to interact. For every point $\mathbf{x}$ in Ω we will define the horizon of $\mathbf{x}$ as the set