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Upscaling of Forchheimer flows

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ABSTRACT

In this work we propose upscaling method for nonlinear Forchheimer flow in heterogeneous porous media. The generalized Forchheimer law is considered for incompressible and slightly-compressible single-phase flows. We use recently developed analytical results (Aulisa et al., 2009) [1] and formulate the resulting system in terms of a degenerate nonlinear flow equation for the pressure with the nonlinearity depending on the pressure gradient. The coarse scale parameters for the steady state problem are determined so that the volumetric average of velocity of the flow in the domain on fine scale and on coarse scale are close. A flow-based coarsening approach is used, where the equivalent permeability tensor is first evaluated following streamline methods for linear cases, and modified in order to take into account the nonlinear effects. Compared to previous works (Garibotti and Peszynska, 2009) [2], (Durlofsky and Karimi-Fard) [3], this approach can be combined with rigorous mathematical upscaling theory for monotone operators, (Efendiev et al., 2004) [4], using our recent theoretical results (Aulisa et al., 2009) [1]. The developed upscaling algorithm for nonlinear steady state problems is effectively used for variety of heterogeneities in the domain of computation. Direct numerical computations for average velocity and productivity index justify the usage of the coarse scale parameters obtained for the special steady state case in the fully transient problem. For nonlinear case analytical upscaling formulas in stratified domain are obtained. Numerical results were compared to these analytical formulas and proved to be highly accurate.

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1. Introduction

In recent years, using near well data, such as core data, engineers have been able to create increasingly complex and detailed geocellular models. This compels taking into account highly heterogeneous geological parameters of reservoirs. Such descriptions typically require a high number of computational cells which is difficult to simulate, e.g., in well optimization problems and history matching. To reduce the computational complexity, some type of coarsening and upscaling procedures are needed. The geological parameters, such as permeability or transmissibility and porosity, should be upscaled for each coarse-grid block.

The variety of approaches for upscaling and multiscale methods of fine scaled geological parameters have been proposed for the linear Darcy case (see [5–10]). These approaches include upscaling methods, see [5,8–10] and multiscale methods [4–7]. In both approaches, a goal is to represent the solution on a coarse grid where each coarse-grid block consists of a union of connected fine-grid blocks. In upscaling methods, the upscaled permeability is calculated in each coarse-grid block by solving local problems with specified boundary conditions and calculating the average of the flux. Local problems can be solved in extended domains for computing the effective properties. In multiscale methods, the local multiscale basis functions are computed instead of local effective properties and these basis functions are coupled via a global formulation.

The extensions of these methods to nonlinear flows, such as Forchheimer flow, are carried out in several papers, see [2,3] which are closely related to our work. In [3], the authors consider the use of iterative upscaling techniques where at each iteration, local–global upscaling technique is used. The work [2] closely relates to our work. In [2], the authors start with a full nonlinear upscaling where the upscaled conductivity is a nonlinear function of the pressure gradient. The starting point of our approach follows [2]. In [2], the authors further use special nonlinear forms for upscaled Forchheimer flows that simplify the upscaling calculations. In the current paper, our goal is to carry out a nonlinear upscaling using

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new formulations of Forchheimer flows. We emphaisize that the monotonicty of the fine-scale operator (as discussed in [2]) is important for formulating full nonlinear homogenization.

In current paper, we utilize recent finding [1], where Forchheimer equation is written in an equivalent form using monotone nonlinear permeability function depending on gradient of pressure. This equivalent formulation reduces the original system of equations for pressure and velocity to one nonlinear parabolic or elliptic equation for pressure only. The ellipticity constant of this equation degenerates as the pressure gradient converges to infinity. The rate of the degeneration is effectively controlled by the order of Forchheimer polynomial and the structure of the coefficients has the important monotonicity properties (see Proposition III.6 and Lemma III.10 from [1]). It allows to prove results on the well-posedness of the initial boundary value problem and apply numerical homogenization theory.

In this paper we present the upscaling algorithm for fluid flow in incompressible media for two types of fluids, incompressible and slightly compressible. Steady state problem for incompressible flow reduces to the degenerate elliptic equation, however the corresponding problem for compressible fluid reduces to time dependent degenerate parabolic equation.

In this paper we first introduce and investigate the upscaling procedure for the time independent problem in case of incompressible fluid. In case of time dependent problem the question one should address is that while the solution is time dependent, the upscaled parameters are time independent for incompressible media. We use the upscaled parameters obtained for steady state case in the time dependent problem. This procedure is justified by the results obtained in our papers [11,12] and the numerical experiment presented in this article. Namely, we will relate the fine scale fully transient solution to the special pseudo steady state (PSS) solution. This solution has a form $At + W(x)$, where A is a constant and $W(x)$ is a solution of auxiliary steady state boundary value problem for the equation with non zero RHS. According to our results in [11,12] under some assumptions the pseudo steady state pressure and velocity serve as pseudo attractors for fully transient pressure and velocity. To upscale the steady state equation we determine the coarse scale porosity and nonlinear permeability, so that the average volumetric velocity of the flow is preserved.

To evaluate the efficiency of the described method in the time dependent case we compare the productivity index (PI) of the well on fine and coarse grids. The PI is inversely proportional to the difference between the average of pressure in the reservoir and on the well. We select the PI as a criteria for the evaluation of the upscaling method as it is widely used by the engineers, see [13–16] and references therein. In the numerical examples we calculate the difference between the values of the PIs on fine and coarse grids. Our numerical results show that the proposed algorithm provides accurate results for different heterogeneities and nonlinearities in steady state case. Resulting transient velocity and PI on coarse scale also provide accurate approximation of corresponding transient parameters on fine scale for heterogeneous fields considered in the paper. Clearly the accuracy of the proposed method depends on heterogeneities as in a single-phase upscaling, [17], i.e., for highly heterogeneous fields, the accuracy of the method will decrease. The main goal of this paper is to propose a method to handle the nonlinearities and, thus, we do not consider highly heterogeneous fields [17].

The paper is structured as follows. In Section 2.1 we introduce g-Forchheimer equations, review their properties and formulate the problem. In Section 2.2 we obtain the form of the coarse scale equation for generalized Forchheimer flow. In Section 2.3 we introduce the special steady state equation which will be used for upscaling for flow of slightly compressible fluid. We then discuss the theoretical results justifying the use of upscaled parameters from steady state problem in time dependent problem. Section 3 is devoted to description of upscaling algorithm. In Section 4 we obtain the explicit analytical upscaling formulas in case of incompressible fluid for nonlinear Forchheimer flow in stratified region. In contrast with the linear case, the formulas for nonlinear case may depend on boundary data, see (42) and (44). In Sections 5.1 and 5.2 we present the numerical results for the incompressible and slightly compressible flows correspondingly. In Section 5.3 we test the usage of the parameters obtained by upscaling steady state equation in transient case. We present the theoretical and numerical result on convergence of transient velocity and PI to corresponding steady state values for upscaled problem.

2. Problem statement and preliminary results

2.1. Generalized Forchheimer equation

Let $\Omega \subset \mathbb{R}^n$ be the flow domain. Darcy equation describes the linear dependence of velocity u on the pressure gradient ∇p

$$u = -\frac{1}{\mu} k(x) \nabla p. \tag{1}$$

Here $k(x)$ is symmetric positive definite permeability tensor, μ is the viscosity of the fluid.

Forchheimer equation, [18], is known to generalize Darcy's equation to take into account inertial terms and has been introduced in the literature in several forms. E.g., see [19, p. 182]

$$\begin{aligned}
&\text{Two term law} : u + \beta(x)\|u\|u = -\frac{1}{\mu}k(x)\nabla p,\\
&\text{Three term law} : u + a_1(x)\|u\|u + a_2(x)\|u\|^2 u = -\frac{1}{\mu}k(x)\nabla p,\\
&\text{Power law} : u + b_1(x)\|u\|^{m-1}u = -\frac{1}{\mu}k(x)\nabla p, \quad 1.6 \leqslant m \leqslant 2.
\end{aligned} \tag{2}$$

From here on $\|\cdot\|$ is the l_2 vector norm. Coefficients $\beta(x), a_1(x), a_2(x), b_1(x)$ and power m are empirical.

All these relations can be written in a compact form as

$$g(\|u\|, x)u = -\frac{1}{\mu} k(x) \nabla p, \tag{3}$$

for some function $g(s,x) \geqslant 0$ for $s \geqslant 0$. We will refer to (3) as *g-Forchheimer* (*momentum*) *equation*. For simplicity from now on we assume the viscosity $\mu = 1$.

To develop rigorous numerical homogenization concepts for Forchheimer flow, we use the results in [1] which allows writing (3) as a monotone relation for ∇p (see Eq. (6) and the discussion after it). Moreover, this allows obtaining the well-posedness results of the corresponding initial boundary value problem and allows estimating the residual error in numerical homogenization because of monotonicity. It was shown in [1] that the monotone relation between velocity and gradient of pressure exists for general functions $g(s,x)$ in the form

$$\begin{aligned}
g(s,x) &= 1 + \sum_{j=1}^{l} a_j(x) s^{\alpha_j}\\
&= 1 + a_1(x)s^{\alpha_1} + a_2(x)s^{\alpha_2} + \cdots + a_l(x)s^{\alpha_l},
\end{aligned} \tag{4}$$

where $l \geqslant 0$, the exponents satisfy $0 < \alpha_j < \alpha_{j+1}$, and the coefficients $a_j(x) \geq 0, j = 1, \ldots, l$. Thus defined function g in (3) includes all the known cases of Forchheimer flow (1) and (2).

In order to make further constructions we rewrite (3) in the equivalent form solving u implicitly in terms of ∇p. To that purpose using the notation $h(s) = sg(s,x) = \|\xi\|$ and $s = h^{-1}(\|\xi\|)$, where $s, \|\xi\| \geqslant 0$, we introduce the function