Journal of Algebra 470 (2017) 289–299

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

On the Gelfand–Kirillov conjecture for the W-algebras attached to the minimal nilpotent orbits

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ARTICLE INFO

Article history:
Received 12 November 2015
Available online 3 September 2016
Communicated by J.T. Stafford

Keywords:
W-algebra
Gelfand–Kirillov conjecture
Noncommutative localization

ABSTRACT

Consider the W-algebra H attached to the minimal nilpotent orbit in a simple Lie algebra $\mathfrak{g}$ over an algebraically closed field of characteristic 0. We show that if an analogue of the Gelfand–Kirillov conjecture holds for such a W-algebra, then it holds for the universal enveloping algebra $\mathrm{U}(\mathfrak{g})$. This, together with a result of A. Premet, implies that the analogue of the Gelfand–Kirillov conjecture fails for some W-algebras attached to the minimal nilpotent orbit in Lie algebras of types B_n $(n \geq 3)$, D_n $(n \geq 4)$, E_6, E_7, E_8, and F_4.

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1. Introduction

Classical works, see for example [4, Theorems 5.1, 5.4], show that any right Nötherian ring has the quotient field which is a noncommutative skew field. In this framework it is natural to ask whether or not such a skew field is isomorphic to a quotient field of a suitable Weyl algebra over a commutative field. A more precise version of this question

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http://dx.doi.org/10.1016/j.jalgebra.2016.08.021
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is known as the Gelfand–Kirillov conjecture: 'Whether or not the quotient field of the universal enveloping algebra of any algebraic Lie algebra is isomorphic to some Weyl skew field?'. In this paper we study a similar question for some W-algebras.

The solution of the original Gelfand–Kirillov conjecture for Lie algebras of type A and some other cases was settled by I. Gelfand and A. Kirillov themselves [6,7] and is positive. A version of this problem for the W-algebras attached to type A Lie algebras was considered in [5], where the authors provide a positive solution of the corresponding problem. We refer the reader to [1] for a more extensive discussion on the Gelfand–Kirillov conjecture.

In his paper [11], A. Premet shows that the Gelfand–Kirillov conjecture fails for $\mathrm{U}(\mathfrak{g})$ if $\mathfrak{g}$ is simple and $\mathfrak{g}$ is not of type A_n, C_n or G_2. Another result of the same author [10] shows that for a simple Lie algebra $\mathfrak{g}$ we have that $\mathrm{U}(\mathfrak{g})$ is "almost equal" to the tensor product of some W-algebra with a suitable Weyl algebra.

The goal of this paper is to modify the result of [10], i.e. to show that the quotient field of $\mathrm{U}(\mathfrak{g})$ is isomorphic to the quotient field of the tensor product of the same W-algebra with a suitable Weyl algebra. This, together with results of [11], implies that the Gelfand–Kirillov conjecture fails for some W-algebras. It worth mentioning that such W-algebras are deeply studied in [10] and explicit generators and relations are known for them.

From now on the base field for all objects is an algebraically closed field $\mathbb{F}$ of characteristic 0.

2. W-algebras

A W-algebra $\mathrm{U}(\mathfrak{g}, e')$ is a finitely generated algebra attached to a semisimple Lie algebra $\mathfrak{g}$ and an $\mathfrak{sl}_2$-triple $\{e', h', f'\}$ inside $\mathfrak{g}$ (see for example [10]). The isomorphism class of such an algebra $\mathrm{U}(\mathfrak{g}, e')$ depends only on the conjugacy class of e'. We are particularly interested in W-algebras attached to an element e' from the minimal nonzero nilpotent orbit in $\mathfrak{g}$. We take an explicit presentation by generators and relations for such a W-algebra [10, Theorem 1.1] and modify the notation of [10] a little. Namely, to a simple Lie algebra $\mathfrak{g}$ we attach a reductive Lie algebra $\mathfrak{g}_{e'}(0)$ with a $\mathfrak{g}_{e'}(0)$-module $\mathfrak{g}(1)$. Then the algebra H (this is a notation of [10] for such W-algebras) would be generated by $\mathfrak{g}_{e'}(0), \mathfrak{g}(1)$ and an additional element C subject to the following relations:

(i) $xy - yx = [x, y]$ for all $x, y \in \mathfrak{g}_{e'}(0)$, where $[x, y]$ is the Lie bracket of $\mathfrak{g}_{e'}(0)$;
(ii) $xy - yx = x \cdot y$ for all $x \in \mathfrak{g}_{e'}(0), y \in \mathfrak{g}(1)$, where $x \cdot y$ is the action operator of the element $x \in \mathfrak{g}_{e'}(0)$ applied to $y \in \mathfrak{g}(1)$;
(iii) C is central in H;
(iv) $xy - yx = \frac{1}{2}(x, y)(C - \Theta_{\mathrm{Cas}} - c_0) + F(x, y)$ where (x, y) denote the skew-symmetric $\mathfrak{g}_{e'}(0)$-invariant bilinear form on $\mathfrak{g}(1)$, Θ_{Cas} is the Casimir element of $\mathrm{U}(\mathfrak{g}_{e'}(0))$, c_0 is a constant depending on $\mathfrak{g}$, $F(x, y)$ is a skew symmetric function on $\mathfrak{g}(1)$ with values in $\mathrm{U}(\mathfrak{g}_{e'}(0))$, see [10, Theorem 1.1].

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