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Palindromic width of wreath products

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ABSTRACT

A palindrome is a word which reads the same left-to-right as right-to-left. We show that the wreath product $G \wr \mathbb{Z}^n$ of any finitely generated group G with $\mathbb{Z}^n$ has finite palindromic width. This generalizes the main result from [16]. We also show that $C \wr A$ has finite palindromic width if C has finite commutator width and A is a finitely generated infinite abelian group. Further we prove that if H is a non-abelian group with finite palindromic width and G any finitely generated group, then every element of the subgroup $G' \wr H$ can be expressed as a product of uniformly boundedly many palindromes. From this we obtain that $P \wr H$ has finite palindromic width if P is a perfect group and further that $G \wr F$ has finite palindromic width for any finite, non-abelian group F.

1. Introduction

Palindromic words in groups have been studied from various angles lately. They make their first appearance in [8], where D. Collins studied palindromic automorphisms of free groups. In [11] H.H. Glover and C.A. Jensen study the geometry of palindromic automorphism groups of the free group. Later in [7] it was shown that free groups have infinite palindromic and primitive width and F. Deloup [9] studied the palindromic map, which is an anti-automorphism, in braid and Artin groups. In Coxeter groups the

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palindromes in the standard generating set are exactly the conjugates of the generators. The property of having finite palindromic width has been studied there under the term *reflection length*. In [15] it has been shown by J. McCammond and T.K. Peterson that affine Coxeter groups have uniformly bounded reflection length. For non-affine Coxeter groups, K. Duszenko [10] proved that the reflection length is infinite by constructing hyperbolic quotients.

More recently, it has been established by V. Bardakov and K. Gongopadhyay [6] that free nilpotent groups and free abelian-by-nilpotent groups have finite palindromic width. A second very recent paper by the same authors shows that some extensions and quotients of these groups have finite palindromic width as well [4]. Another paper [5] by the same authors proves that certain soluble groups have finite palindromic width. In [6] they use results about the commutator width in nilpotent groups to establish their result. Independently, it has been shown by T. Riley and A. Sale in [16] that free metabelian groups have finite palindromic width by using results about skew-symmetric functions on free abelian groups. Further, the same authors show that $B \wr \mathbb{Z}^n$ has finite palindromic width if B is a group with finite palindromic width.

We extend this result to the case where G is any finitely generated group, then we show that $G \wr \mathbb{Z}^n$ has finite palindromic width. A result by M. Akhavan-Malayeri [3] shows that the wreath product of F_d with $\mathbb{Z}^n$ has finite commutator width. We use the result from [3] to prove that $F_d \wr \mathbb{Z}^n$ has finite palindromic width and then deduce that this property also holds for its quotients. We also give a proof that the wreath product $C \wr A$ has finite palindromic width if C is a finitely generated group that has finite commutator width and A a finitely generated infinite abelian group.

More generally, let G be any finitely generated group and H a non-abelian group which has finite palindromic width with respect to some generating set. We establish that every element of the subgroup $G' \wr H$ of the regular wreath product $G \wr H$ is a finite product of palindromes. As a corollary we obtain that $G \wr F$ has finite palindromic width if F is a non-abelian finite group or if G is perfect.

More concretely, we prove the following, where $pw(G, X)$ denotes the palindromic width of the group G with respect to the generating set X.

Theorem 1.1.

1. *Let G be a d-generated group generated by X and E be the standard generating set of $\mathbb{Z}^k$, for $k \in \mathbb{N}$. Then we have that $pw\left(G \wr \mathbb{Z}^k, X \cup E\right) \leq 4d + 8k - 1$ if k is even and $pw\left(G \wr \mathbb{Z}^k, X \cup E\right) \leq 4d + 8k + 1$ if k is odd.*
2. *Assume that A is an r-generated infinite abelian group generated by T and C a finitely generated group with finite commutator width n. Then $pw(C \wr A, T \cup Y) \leq 4|Y| + 5r + 7n$, for any finite generating set Y of C.*
3. *Let G be a finitely generated group and H a non-abelian finitely generated group with finite palindromic width with respect to the generating set Z such that there exists a relation w that holds in H but $\overline{w} \neq 1$. Then every element of the subgroup $G' \wr H$ can be written as a product of at most $pw(H, Z) + 1$ palindromes.*