Journal of Algebra 427 (2015) 118–141

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Grade, dominant dimension and Gorenstein algebras

Nan Gao [a,1], Steffen Koenig [b,*]

[a] Department of Mathematics, Shanghai University, Shanghai 200444, PR China
[b] Institut für Algebra und Zahlentheorie, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany

ARTICLE INFO

Article history:
Received 19 May 2014
Available online 14 January 2015
Communicated by Michel Broué

Keywords:
Grade
Double centraliser property
Dominant dimension
Gendo-Gorenstein algebra
Gorenstein projective module
Maximal orthogonal subcategory

ABSTRACT

We first give precise connections between Auslander–Bridger's grade, double centraliser properties and dominant dimension, and apply these to homological conjectures. Then we introduce gendo-d-Gorenstein algebras as correspondents of Gorenstein algebras under a Morita–Tachikawa correspondence. We characterise these algebras by homological properties and derive several of their properties, including higher Auslander correspondence.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

A classical result of representation theory is the Morita–Tachikawa correspondence [23] providing a bijection between pairs (Λ an artin algebra, M a generator-cogenerator) and algebras Γ of dominant dimension at least two. A celebrated special case is Auslander's bijection relating algebras Λ of finite representation type with algebras Γ of global dimension at most two and dominant dimension at least two. This has been generalised by Iyama [16] to 'Higher Auslander correspondence'. More recently, other

* Corresponding author.
E-mail addresses: nangao@shu.edu.cn (N. Gao), skoenig@mathematik.uni-stuttgart.de (S. Koenig).
[1] Supported by the National Natural Science Foundation of China (Grant No. 11101259).

http://dx.doi.org/10.1016/j.jalgebra.2014.11.028
0021-8693/© 2014 Elsevier Inc. All rights reserved.

interesting classes of algebras have been defined by specialising the Morita–Tachikawa correspondence: Gendo-symmetric algebras [11,12] correspond to pairs (Λ, M) where Λ is symmetric; Morita algebras [19] correspond to pairs where Λ is self-injective.

In this paper, another such class of artin algebras is introduced, corresponding to pairs where Λ is d-Gorenstein and M is a Gorenstein projective generator. These 'gendo-d-Gorenstein algebras' Γ will be given two different characterisations, in terms of homological properties (see Theorem 4.2). Both characterisations explain $d = \mathrm{injdim}_\Lambda \Lambda$ in terms of Γ. A number of properties of these algebras is established, concerning global dimension, Hochschild cohomology, higher Auslander correspondence, and more.

In order to prove these results, a close relation between dominant dimension and double centraliser properties on the one hand and Auslander–Bridger's concept of grade is being worked out, extending results of Buchweitz [8]. The results give precise connections between these concepts, valid for artin algebras in general. As a by-product, some new results about homological conjectures are obtained.

This article is organised as follows: Section 2 is devoted to clarifying the connections among grade, double centraliser properties and dominant dimension for artin algebras. The main results are Theorem 2.3 characterising the double centraliser property in terms of grade, and Theorem 2.14 characterising grade in terms of dominant dimension. Section 3 uses the techniques set up in Section 2 to prove some assertions about homological conjectures, in particular several sufficient criteria for a 'grade' version of the Strong Nakayama Conjecture to hold true.

In Section 4, gendo-d-Gorenstein algebras are defined as correspondents of Gorenstein algebras under a Morita–Tachikawa correspondence, and characterised (in Theorem 4.2) in terms of homological properties. The final Section 5 then provides properties of these algebras. The proofs strongly use the techniques provided in Section 2.

2. Grade and dominant dimension

In this section, we will relate grade to double centraliser properties and to dominant dimension. By building on [8] we clarify the connections between these concepts.

Let Λ be an artin algebra. Denote by Λ-mod the category of finitely generated left Λ-modules. Recall from [26] that the *dominant dimension* of a module M in Λ-mod, which we denote by $\mathrm{domdim}\, M$, is the maximal number t (or ∞) having the following property: let $0 \to M \to I^0 \to I^1 \to \cdots \to I^t \to \cdots$ be a minimal injective resolution of M, then I^j is projective for all $j < t$ (or ∞). Let e be an idempotent of Λ and Λe a faithful projective-injective left Λ-module. Then $\mathrm{domdim}\, \Lambda \geq 2$ if and only if the left Λ-module Λe has the double centraliser property which means that $\mathrm{End}_{e\Lambda e}(\Lambda e) \cong \Lambda$ (see [26, 7.7]). Buchweitz [8, Proposition 2.9] showed for any idempotent e that $\Lambda \cong \mathrm{End}_{e\Lambda e}(\Lambda e)$ if and only if Auslander–Bridger's grade of $\Lambda/\Lambda e\Lambda$ as a right Λ-module is greater than 2.

Let M and T be in Λ-mod. Extending the classical theory of dominant dimension, one defines the *dominant dimension of* T *relative to* M, M-$\mathrm{domdim}\, T$, as the supremum of all $n \in \mathbb{Z}$ such that there exists an exact sequence $0 \to T \to M^1 \to M^2 \to \cdots \to M^n$ with all

Download English Version:

https://daneshyari.com/en/article/4584505

Download Persian Version:

https://daneshyari.com/article/4584505

Daneshyari.com