Journal of Functional Analysis 269 (2015) 3765–3811

Contents lists available at ScienceDirect

Journal of Functional Analysis

www.elsevier.com/locate/jfa

# Functional central limit theorem for Brownian particles in domains with Robin boundary condition ☆

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A R T I C L E  I N F O

*Article history:*
Received 13 June 2014
Accepted 27 September 2015
Available online 21 October 2015
Communicated by L. Saloff-Coste

*MSC:*
primary 60F17
secondary 60H15

*Keywords:*
Fluctuation
Hydrodynamic limit
Robin boundary condition
Stochastic partial differential equation

A B S T R A C T

We rigorously derive non-equilibrium space–time fluctuation for the particle density of a system of reflected diffusions in bounded Lipschitz domains in $\mathbb{R}^d$. The particles are independent and are killed by a time-dependent potential which is asymptotically proportional to the boundary local time. We generalize the functional analytic framework introduced by Kotelenez [20,21] to deal with time-dependent perturbations. Our proof relies on Dirichlet form method rather than the machineries derived from Kotelenez's sub-martingale inequality. Our result holds for any symmetric reflected diffusion, for any bounded Lipschitz domain and for any dimension $d \geq 1$.



**Contents**



☆ Research partially supported by NSF Grants DMS-1206276 and DMR-1035196.

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http://dx.doi.org/10.1016/j.jfa.2015.09.022

## 1. Introduction

The goal of this paper is to develop a machinery to overcome some difficulties that arise in the study of fluctuations for systems of reflected diffusions (such as reflected Brownian motions) with a singular type of time-dependent killing potential. The primary examples are the systems of annihilating diffusions introduced in [4] and [5], which can be used to model the transport of positive and negative charges in solar cells or the population dynamics of two segregated species under competition. The model in [5] consists of two families of reflected diffusions confined in two adjacent domains, say two adjacent rectangles $(0,2)\times(0,1)$ and $(0,2)\times(-1,0)$, respectively. These two families of particles (positive and negative charges respectively) annihilate each other at a certain rate when they come close to each other near the interface $(0,2)\times\{0\}$. This interaction models the annihilation, trapping, recombination and separation phenomena of the charges. From the viewpoint of the positive charges, they are themselves reflected diffusions in $(0,2)\times(0,1)$ subject to killing by a time-dependent random potential.

In this paper, we focus our attention to a one-type particle model which consists of i.i.d. reflected diffusions killed by a deterministic time-dependent potential near the boundary. The following assumption on reflected diffusions is in force throughout this paper:

**Assumption 1.1.** Suppose $D\subset\mathbb{R}^d$ is a bounded Lipschitz domain, $\rho\in W^{1,2}(D)\cap C(\overline{D})$ is a strictly positive function, $\mathbf{a}=(a^{ij})$ is a symmetric, bounded, uniformly elliptic $d\times d$ matrix-valued function such that $a^{ij}\in W^{1,2}(D)$ for each $i$, $j$. Here $C(\overline{D})$ denotes the