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Functional central limit theorem for Brownian particles in domains with Robin boundary condition [☆]



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ABSTRACT

We rigorously derive non-equilibrium space–time fluctuation for the particle density of a system of reflected diffusions in bounded Lipschitz domains in \mathbb{R}^d . The particles are independent and are killed by a time-dependent potential which is asymptotically proportional to the boundary local time. We generalize the functional analytic framework introduced by Kotelenetz [20,21] to deal with time-dependent perturbations. Our proof relies on Dirichlet form method rather than the machineries derived from Kotelenetz’s sub-martingale inequality. Our result holds for any symmetric reflected diffusion, for any bounded Lipschitz domain and for any dimension $d \geq 1$.

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1. Introduction

The goal of this paper is to develop a machinery to overcome some difficulties that arise in the study of fluctuations for systems of reflected diffusions (such as reflected Brownian motions) with a singular type of time-dependent killing potential. The primary examples are the systems of annihilating diffusions introduced in [4] and [5], which can be used to model the transport of positive and negative charges in solar cells or the population dynamics of two segregated species under competition. The model in [5] consists of two families of reflected diffusions confined in two adjacent domains, say two adjacent rectangles $(0, 2) \times (0, 1)$ and $(0, 2) \times (-1, 0)$, respectively. These two families of particles (positive and negative charges respectively) annihilate each other at a certain rate when they come close to each other near the interface $(0, 2) \times \{0\}$. This interaction models the annihilation, trapping, recombination and separation phenomena of the charges. From the viewpoint of the positive charges, they are themselves reflected diffusions in $(0, 2) \times (0, 1)$ subject to killing by a time-dependent random potential.

In this paper, we focus our attention to a one-type particle model which consists of i.i.d. reflected diffusions killed by a deterministic time-dependent potential near the boundary. The following assumption on reflected diffusions is in force throughout this paper:

Assumption 1.1. Suppose $D \subset \mathbb{R}^d$ is a bounded Lipschitz domain, $\rho \in W^{1,2}(D) \cap C(\overline{D})$ is a strictly positive function, $\mathbf{a} = (a^{ij})$ is a symmetric, bounded, uniformly elliptic $d \times d$ matrix-valued function such that $a^{ij} \in W^{1,2}(D)$ for each i, j . Here $C(\overline{D})$ denotes the

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