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On parabolic induction associated with a p -adic symmetric space



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ABSTRACT

We provide some tools to study distinguished induced representations in the setting of a general p -adic symmetric space.

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1. Introduction

Let G be a reductive p -adic group, θ an involution on G and $H = G^\theta$. Fix a character χ of H . A (smooth, complex valued) representation π of G is (H, χ) -distinguished if $\text{Hom}_H(\pi, \chi) \neq 0$.

This note examines the relation between distinction of an induced representation and distinction of the inducing data (or, more precisely, of some Jacquet module of the inducing data) in the setting of any p -adic symmetric space. Our main result is a generalization to this setting of a necessary condition for an induced representation of G to be (H, χ) -distinguished in terms of distinction of some Jacquet module of the inducing data.

The result we have in mind proved to be a useful tool in the study of distinction problems in the special cases where it was already proved (see [Off06,FLO12,Gur15]). It further led to applications in the study of period integrals of automorphic forms. The relevant results were proved separately in each case.

For future reference, it will be of use to have the results available in a general setting. In particular, the results of this paper will be applied to the study of distinguished representations of classical groups. A particular case of study, also related to the descent construction of Ginzburg, Rallis and Soudry (see e.g. [GRS11]), is the case where $G = U_{2n}(E/F)$ is a unitary group associated with the quadratic extension E/F and $H = \text{Sp}_{2n}(F)$. In a work in progress joint with Arnab Mitra we study distinguished representations in this setting. Combining ideas of Ash, Ginzburg and Rallis ([AGR93]) with the results of this note we can already show the following result. If an irreducible representation of G is H -distinguished, then its cuspidal support is a representation of a Levi subgroup that is contained in the Siegel Levi. We expect the tools developed here to have many further application for this and other symmetric spaces.

We consider the restriction to H of a parabolically induced representation of G . Our main tool is a combination of the geometric lemma of Bernstein and Zelevinsky [BZ77] and a careful study of parabolic orbits on the symmetric space G/H , where we generalize results of Lapid and Rogawski [LR03].

Our main result can be formulated as follows (see Theorem 4.2 for a more notationally involved formulation). Fix a minimal parabolic subgroup P_0 of G and let δ_A be the modulus function of a locally compact group A .

Theorem 1.1. *Let $P \supseteq P_0$ be a parabolic subgroup of G with standard Levi subgroup M , σ a representation of M , $\pi = \text{Ind}_P^G(\sigma)$ the normalized parabolic induction and χ a character of H . If π is (H, χ) -distinguished then there exists $\eta \in G$ so that the following conditions are satisfied:*

- *The group $L = M \cap \eta\theta(\eta^{-1}M\eta)\eta^{-1}$ is the standard Levi subgroup of a parabolic $Q \supseteq P_0$ of G contained in P .*

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