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On the Laplacian spectra of some variants of corona

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A B S T R A C T

Among many graph operations corona of graphs is one of the well known graph operation which fascinates many researchers because of its beautiful graph structure. Subsequently, many variants of corona operation are defined and the spectral properties of these graphs have been studied. In this paper, we define two special forms of matrices namely: super corona matrix and super neighbourhood corona matrix. We describe all the eigenvalues and the corresponding eigenvectors of these matrices. Further, we define some more variants of corona graphs such as subdivision double corona, Q-graph double corona, R-graph double corona, total double corona, subdivision double neighbourhood corona, Q-graph double neighbourhood corona, R-graph double neighbourhood corona and total double neighbourhood corona. We give a complete description of the eigenvalues and the eigenvectors of graphs obtained under such operations with the help of the results obtained for super corona and super neighbourhood corona matrices.

1. Introduction

Throughout this paper we consider only simple graphs. Let $G = (V, E)$ be a graph with vertex set $V = \{1, 2, \ldots, n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$. Let $M(G)$ be the

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vertex-edge incidence matrix of G and $A(G)$ be the adjacency matrix of G. The Laplacian matrix (denoted by $L(G)$) and the signless Laplacian matrix (denoted by $|L|(G)$) of G are defined as $L(G) = D(G) - A(G)$ and $|L|(G) = D(G) + A(G)$, respectively, where $D(G)$ is the diagonal matrix with main entries as vertex degrees of G. Let $\lambda_1(G) \leq \lambda_2(G) \leq \ldots \leq \lambda_n(G)$, $\mu_1(G) \leq \mu_2(G) \leq \ldots \leq \mu_n(G)$ and $\nu_1(G) \leq \nu_2(G) \leq \ldots \leq \nu_n(G)$ denote the eigenvalues of $L(G)$, $A(G)$ and $|L|(G)$, respectively. The collection of all the eigenvalues of a matrix together with its multiplicities is known as the spectrum of that matrix. So depending upon which matrix representation of a graph is considered, we have the Laplacian spectrum, the adjacency spectrum (commonly known as spectrum) and the signless Laplacian spectrum of the graph. Two nonisomorphic graphs are said to be Laplacian (adjacency, signless Laplacian) cospectral if their Laplacian (adjacency, signless Laplacian) spectra are same. It is well known that the spectrum of a graph contains a lot of structural information about the graph; see for example [1,4,6] and the references therein.

It is known that the smallest Laplacian eigenvalue of G, $\lambda_1(G)$ is always 0 and the corresponding eigenvector is $\mathbf{1}$, the vector of all ones. Fiedler [7] showed that the second smallest Laplacian eigenvalue of G, $\lambda_2(G)$ is positive if and only if G is connected. Thus $\lambda_2(G)$ is known as the *algebraic connectivity* of G, denoted by $a(G)$. The eigenvectors corresponding to $a(G)$ are popularly known as *Fiedler vectors*. For more on Laplacian spectrum of graphs follow the survey paper by Merris [16] and the references therein.

Many graph operations such as disjoint union, complement, products (Cartesian, direct, strong, lexicographic product etc.), join, corona, edge corona, neighbourhood corona, variants of corona (subdivision vertex corona, subdivision edge corona, R-vertex corona, R-edge corona) and variants of neighbourhood corona (subdivision vertex neighbourhood corona, subdivision edge neighbourhood corona, R-vertex neighbourhood corona, R-edge neighbourhood corona) are introduced and their spectra are described in [2–6,8–15,17]. Frucht and Harary first introduced the corona operation to construct a graph whose automorphism group is the wreath product of the two component automorphism groups. Then attracted by the structure of corona graphs, many researchers defined successively many variants of it and described their spectra (Laplacian, signless Laplacian spectra as well) in terms of the spectra of the constituting graphs. The following definitions are taken from [6] which are required to define our new graph operations.

Let G be a connected graph on n vertices and m edges. The *subdivision graph* $S(G)$ of G is the graph obtained by inserting a new vertex into every edge of G. The $Q(G)$*-graph* of G is the graph obtained from G by inserting a new vertex into every edge of G and by joining by edges those pairs of these new vertices which lie on adjacent edges of G. The $R(G)$*-graph* of G is defined as the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end points of the edge corresponding to it. The *total graph* of G, denoted by $T(G)$, is the graph whose set of vertices is the union of the set of vertices and set of edges of G, with two vertices of $T(G)$ being adjacent if and only if the corresponding elements of G are adjacent or incident. Note that $S(G)$, $Q(G)$, $R(G)$ and $T(G)$ have $n + m$ vertices each. In the above