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Brauer-type eigenvalue inclusion sets and the spectral radius of tensors

Changjiang Bu [a,b,*], Xiuquan Jin [a], Haifeng Li [b], Chunli Deng [b]

[a] *College of Science, Harbin Engineering University, Harbin 150001, PR China*
[b] *College of Automation, Harbin Engineering University, Harbin 150001, PR China*

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In this paper, we give two Brauer-type eigenvalue inclusion sets and some bounds on the spectral radius for tensors. As applications, some bounds on the spectral radius of uniform hypergraphs are presented.

1. Introduction

For a positive integer n, let $[n] = \{1, 2, \dots, n\}$. An order m dimension n tensor $\mathcal{A} = (a_{i_1 i_2 \cdots i_m})$ is a multidimensional array with n^m complex entries, where $i_j \in [n]$, $j \in [m]$. Let $\mathbb{C}^{[m,n]}$ denote the set of order m dimension n tensors over complex field $\mathbb{C}$, and $\mathbb{C}^n$ be the set of dimension n column vectors over $\mathbb{C}$.

* Corresponding author.
E-mail address: buchangjiang@hrbeu.edu.cn (C. Bu).

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In 2005, Qi [1] and Lim [2] introduced the notion of eigenvalue for tensors, independently. For $\mathcal{A} = (a_{i_1 i_2 \cdots i_m}) \in \mathbb{C}^{[m,n]}$, $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in \mathbb{C}^n$, $\mathcal{A}x^{m-1}$ is a dimension n column vector with entries

$$(\mathcal{A}x^{m-1})_i = \sum_{i_2,\ldots,i_m=1}^{n} a_{ii_2\cdots i_m} x_{i_2} \cdots x_{i_m},\ i \in [n].$$

If there exists a nonzero vector $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in \mathbb{C}^n$ and a number $\lambda \in \mathbb{C}$ such that

$$\mathcal{A}x^{m-1} = \lambda x^{[m-1]}, \tag{1}$$

then λ is called an eigenvalue of $\mathcal{A}$, x is called an eigenvector of $\mathcal{A}$ corresponding to λ, where $x^{[m-1]} = (x_1^{m-1}, \ldots, x_n^{m-1})^{\mathrm{T}}$. Let $\sigma(\mathcal{A})$ denote the set of all eigenvalues of $\mathcal{A}$, $\rho(\mathcal{A}) = \max\{|\lambda| : \lambda \in \sigma(\mathcal{A})\}$ be the spectral radius of $\mathcal{A}$, $r_i(\mathcal{A}) = \sum\limits_{i_2,\ldots,i_m=1}^{n} |a_{ii_2\cdots i_m}|$, $R_i(\mathcal{A}) = r_i(\mathcal{A}) - |a_{ii\cdots i}|$.

Let $\mathcal{H} = (V(\mathcal{H}), E(\mathcal{H}))$ be a hypergraph with vertex set $V(\mathcal{H})$ and edge set $E(\mathcal{H})$. If every edge of $\mathcal{H}$ contains exactly k distinct vertices, then $\mathcal{H}$ is called a k-uniform hypergraph. The degree of a vertex i in $\mathcal{H}$ is the number of edges incident with i, denoted by d_i. In 2012, Cooper et al. [15] gave the concept of the adjacency tensor of a uniform hypergraph. For a k-uniform hypergraph $\mathcal{H}$, the adjacency tensor $\mathcal{A}_{\mathcal{H}} = (a_{i_1 i_2 \cdots i_k})$ is an order k dimension $|V(\mathcal{H})|$ tensor with entries

$$a_{i_1 i_2 \cdots i_k} = \begin{cases} \frac{1}{(k-1)!}, & \{i_1, i_2, \ldots, i_k\} \in E(\mathcal{H}), \\ 0, & \textit{otherwise}. \end{cases}$$

Eigenvalues of $\mathcal{A}_{\mathcal{H}}$ are called eigenvalues of $\mathcal{H}$, the spectral radius of $\mathcal{A}_{\mathcal{H}}$ is called the spectral radius of $\mathcal{H}$, denoted by $\rho(\mathcal{H})$.

In recent years, the spectral theory of tensors and spectral hypergraph theory have attracted much attention. In 2005, Qi [1] gave the Geršgorin-type eigenvalue inclusion sets for real symmetric tensor $\mathcal{A} = (a_{i_1 i_2 \cdots i_m})$ as follows:

$$\sigma(\mathcal{A}) \subseteq \mathcal{G}(\mathcal{A}) = \bigcup_{i=1}^{n} \{z \in \mathbb{C} : |z - a_{i\cdots i}| \leq R_i(\mathcal{A})\}, \tag{2}$$

this result also holds for $\mathcal{A} \in \mathbb{C}^{[m,n]}$ (see [10]). In 2015, Bu et al. [14] established the Brualdi-type eigenvalue inclusion sets of tensors via a digraph. In 2016, Li et al. [9] gave a variation of Brauer-type eigenvalue inclusion sets for tensors. In addition, many researchers are interested in the spectral radius of nonnegative tensors, and give various bounds to estimate the spectral radius (see [10–13]). For a k-uniform hypergraph $\mathcal{H}$ with maximum degree Δ, Yuan et al. [16] gave the bounds on $\rho(\mathcal{H})$ via degrees of vertices, Lin et al. [17] showed upper bounds for $\rho(\mathcal{H})$ in terms of the average 2-degrees of vertices, Li et al. [18] obtained the bounds of $\Delta - \rho(\mathcal{H})$ in terms of some graph parameters.