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Distance spectral radius of uniform hypergraphs

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A R T I C L E I N F O

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We study the effect of three types of graft transformations to increase or decrease the distance spectral radius of connected uniform hypergraphs, and we determine the unique k-uniform hypertrees with maximum, second maximum, minimum and second minimum distance spectral radius, respectively.

1. Introduction

A hypergraph G consists of a vertex set $V(G)$ and an edge set $E(G)$, where $V(G)$ is nonempty, and each edge $e \in E(G)$ is a nonempty subset of $V(G)$, see [2]. The order of G is $|V(G)|$. For an integer $k \geq 2$, we say that a hypergraph G is k-uniform if every edge has size k. A (simple) graph is a 2-uniform hypergraph. The degree of a vertex v in G, denoted by $d_G(v)$, is the number of edges of G which contain v.

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For $u, v \in V(G)$, a walk from u to v in G is defined to be a sequence of vertices and edges $(v_0, e_1, v_1, \dots, v_{p-1}, e_p, v_p)$ with $v_0 = u$ and $v_p = v$ such that edge e_i contains vertices v_{i-1} and v_i, and $v_{i-1} \neq v_i$ for $i = 1, \dots, p$. The value p is the length of this walk. A path is a walk with all v_i distinct and all e_i distinct. A cycle is a walk containing at least two edges, all e_i are distinct and all v_i are distinct except $v_0 = v_p$. A vertex $u \in V(G)$ is viewed as a path (from u to u) of length 0. If there is a path from u to v for any $u, v \in V(G)$, then we say that G is connected. A hypertree is a connected hypergraph with no cycles. Note that a k-uniform hypertree with m edges always has order $1 + (k-1)m$.

Let G be a connected k-uniform hypergraph with $V(G) = \{v_1, \dots, v_n\}$. For $u, v \in V(G)$, the distance between u and v is the length of a shortest path from u to v in G, denoted by $d_G(u, v)$. In particular, $d_G(u, u) = 0$. The diameter of G is the maximum distance between all vertex pairs of G. The distance matrix of G is the $n \times n$ matrix $D(G) = (d_G(u, v))_{u,v \in V(G)}$. The eigenvalues of $D(G)$ are called the distance eigenvalues of G. Since $D(G)$ is real and symmetric, the distance eigenvalues of G are real. The distance spectral radius of G, denoted by $\rho(G)$, is the largest distance eigenvalue of G. Note that $D(G)$ is an irreducible nonnegative matrix. The Perron–Frobenius theorem implies that $\rho(G)$ is simple, and there is a unique positive unit eigenvector corresponding to $\rho(G)$, which is called the distance Perron vector of G, denoted by $x(G)$.

The study of distance eigenvalues of 2-uniform hypergraphs (ordinary graphs) dates back to the classical work of Graham and Pollack [5], Graham and Lovász [4] and Edelberg et al. [3]. Ruzieh and Powers [7] showed that among connected 2-uniform hypergraphs of order n, the path P_n is the unique graph with maximum distance spectral radius. Stevanović and Ilić [9] showed that among trees of order n, the star S_n is the unique tree with minimum distance spectral radius. Nath and Paul [6] determined the unique trees with maximum distance spectral radius among trees with fixed matching number. For more details on distance eigenvalues and especially on distance spectral radius of 2-uniform hypergraphs, one may refer to the recent survey of Aouchiche and Hansen [1] and references therein. Sivasubramanian [8] gave a formula for the inverse of a few q-analogs of the distance matrix of a 3-uniform hypertree.

For a k-uniform hypertree G with $V(G) = \{v_1, \dots, v_n\}$, if $E(G) = \{e_1, \dots, e_m\}$, where $e_i = \{v_{(i-1)(k-1)+1}, \dots, v_{(i-1)(k-1)+k}\}$ for $i = 1, \dots, m$, then we call G a k-uniform loose path, denoted by $P_{n,k}$.

For a k-uniform hypertree G of order n, if there is a partition of the vertex set $V(G)$ into $\{u\} \cup V_1 \cup \dots \cup V_m$ such that $|V_1| = \dots = |V_m| = k - 1$, and $E(G) = \{\{u\} \cup V_i : 1 \leq i \leq m\}$, then we call G is a (k-uniform) hyperstar (with center u), denoted by $S_{n,k}$. In particular, $S_{1,k}$ is a hypergraph with a single vertex and $S_{k,k}$ is a k-uniform hypergraph with a single edge.

In this paper, we study the effect of three types of graft transformations to increase or decrease the distance spectral radius of connected k-uniform hypergraphs. As applications, we show that $P_{n,k}$ and $S_{n,k}$ are the unique k-uniform hypertrees with maximum and minimum distance spectral radius, respectively, and we also determine the unique