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Generalized inverse eigenvalue problem for matrices whose graph is a path

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ABSTRACT

In this paper, we analyse a special generalized inverse eigenvalue problem $A_n x = \lambda B_n x$ for the pair (A_n, B_n) of matrices each of whose graph is a path on n vertices, by investigating the leading principal minors of the matrix $A_n - \lambda B_n$. From the given data consisting of B_n, a matrix A_k $(k < n)$ whose graph is a path on k vertices, two column vectors $X_2, Y_2 \in \mathbb{R}^{n-k}$ and distinct real numbers λ and μ we construct A_n and two column vectors $X_1, Y_1 \in \mathbb{R}^k$ such that A_k is the leading principal sub-matrix of A_n and (λ, X), (μ, Y) are the eigenpairs of (A_n, B_n), where $X = (X_1^T, X_2^T)^T$ and $Y = (Y_1^T, Y_2^T)^T$. Further, numerical examples are also given to demonstrate the applicability of the results developed here.

1. Introduction

A generalized inverse eigenvalue problem (GIEP) concerns the reconstruction of matrices from a prescribed set of eigen data. The eigen data involved may consist of the complete or only partial information of eigenvalues or eigenvectors. Together with the eigen data some restrictions may be imposed on the structure of the involved matrices. One such restriction is the zero pattern of the matrices, i.e. the GIEP may require the matrices to maintain certain pre-determined pattern of the positions of the zero entries.

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A well known way of assigning the zero pattern is to specify the graphs of the involved matrices. The GIEP for a pair (A, B) of $n \times n$ matrices involves the generalized eigenvalue problem of the form $Ax = \lambda Bx$. When $B = I$, the identity matrix, the problem reduces to the standard form, which has a quite well-developed theory.

A detailed analysis of the general eigenvalue problems has been done in [1]. An extensive characterization of the inverse eigenvalue problems is found in [2]. Some special types of inverse eigenvalue problems have been studied in [3–6]. IEPs and GIEPs involving Jacobi matrices have been studied in [7–9]. Inverse problems involving matrices with prescribed graph have been studied in [10–13]. Inverse eigenvalue problems arise in a wide variety of applications such as control theory, pole assignment problems, system identification, structural analysis, mass spring vibrations, circuit theory, mechanical system simulation and graph theory [1,2,13,14]. In various computations involving a complicated physical system it is often impossible to obtain the entire spectral information [15] and so it is necessary to consider a partially described problem where only a portion of eigenvalues and eigenvectors is prescribed. In this paper we consider a special GIEP with partial eigen data for a pair of matrices whose graph is a path on n vertices.

Given an $n \times n$ symmetric matrix A, the graph of A, denoted by $G(A)$, has vertex set $V = \{1, 2, 3, \dots, n\}$ and edges $\{i, j\}$, $i \neq j$ if and only if $a_{ij} \neq 0$. Also, for a graph G, we denote by $S(G)$ the set of all symmetric matrices which have G as their graph. The path on n vertices is a graph with vertex set $\{1, 2, \dots, n\}$ and with edges $\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \dots, \{n-1, n\}\}$. It is denoted by P_n. So for any $A \in S(P_n)$, we have $a_{i,i+1} = a_{i+1,i} \neq 0$, $i = 1, 2, \dots, n-1$. Thus $S(P_n)$ consists of tridiagonal matrices with non-zero off-diagonal entries, provided the vertices are labelled according as they appear in one traversal of the path.

The following notations will be used throughout this paper:

(i) P_n will denote a path on n vertices $\{1, 2, 3, \dots, n\}$. P_k, $1 \leqslant k < n$, will denote the path obtained from P_n by retaining the first k vertices.

(ii) A_n and B_n will denote matrices of $S(P_n)$, of the following forms:

$$A_n = \begin{pmatrix} a_1 & -b_1 & 0 & \dots & 0 & 0 \\ -b_1 & a_2 & -b_2 & \dots & 0 & 0 \\ 0 & -b_2 & a_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_{n-1} & -b_{n-1} \\ 0 & 0 & 0 & \dots & -b_{n-1} & a_n \end{pmatrix},$$

$$B_n = \begin{pmatrix} c_1 & d_1 & 0 & \dots & 0 & 0 \\ d_1 & c_2 & d_2 & \dots & 0 & 0 \\ 0 & d_2 & c_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & c_{n-1} & d_{n-1} \\ 0 & 0 & 0 & \dots & d_{n-1} & c_n \end{pmatrix}$$